Simulation of melting of ice in a porous media under multiple constant temperature heat sources using a combined transfinite interpolation and PDE methods

P. Rattanadecho

Faculty of Engineering, Thammasat University (Rangsit Campus), Klong Luang Pathumthani 12121, Thailand

Received 17 December 2005; received in revised form 7 February 2006; accepted 10 February 2006

Available online 6 March 2006

Abstract

A numerical study is made of the melting of ice in a rectangular cavity filled with a porous medium subjected to multiple constant temperature heat sources. Focus is placed on establishing a computationally efficient approach for solving moving boundary heat transfer problem in a two-dimensional structured grids. Specific application to multidimensional melting problem with a complicated moving boundary condition is considered. Preliminary grids are first generated by an algebraic method, based on a transfinite interpolation method, with subsequent refinement using a PDE mapping (parabolic grid generation) method. A preliminary case study indicates successful implementation of the numerical procedure. A two-dimensional melting model is then validated against available analytical solution and experimental results and subsequently used as a tool for efficient computational prototyping.

Keywords: Melting; Porous media; Transfinite interpolation; Moving boundary

1. Introduction

Transient heat transfer problems involving melting or solidification are generally referred to as “phase change” or “moving boundary” problems. They are important topics which span a broad spectrum of scientific and engineering disciplines such as the freezing or thawing of soil, ice formation, crystal growth, aerodynamic ablation, casting of metal, food processing and numerous others. Generally, the solution of moving boundary problem with phase transition has been of special interest due to the inherent difficulties associated with the nonlinearity of the interface conditions and the unknown locations of the arbitrary moving boundaries. Reviews of these problems are available (Murray and Landis, 1959; Hashemi and Sliepecevich, 1973; Frivik and Comini, 1982; Sparrow and Broadbent, 1983; Weaver and Viskanta, 1986; Chellaiah and Viskanta, 1988; Hasan et al., 1991; Charn-Jung and Kaviany, 1992).

In the past, a variety of conventional numerical techniques have been developed for solving these problems, including the enthalpy (Shamsundar and Sparrow, 1976; Crowley, 1978), apparent heat capacity (Bonacina et al., 1973), isotherm migration (Crank and Gupta, 1975), and coordinate transformation methods (Hsu et al., 1981; Sparrow and Chuck, 1984; Sparrow et al., 1978; Cheung et al., 1984; Rattanadecho, 2004a,b). These methods have been introduced by researchers mainly to overcome the difficulties in handling moving boundaries. Previous works on multidimensional moving boundary problems include Duda et al. (1975), Saitoh (1978), Gong and Mujumdar (1998), Cao et al. (1999), Khillarkar et al. (2000), Chatterjee and Prasad (2000) and Beckett et al. (2001).

Conventionally numerical methods have been widely used due to easy to handle numerical algorithms for phase change problem. However, in numerical approximations used in this method with discontinuous coefficients, often the largest numerical errors are introduced in a neighborhood of the discontinuities particularly for phase change in geometry complexity as well as boundary condition.
The troublesome numerical errors in conventional method are effectively reduced if the grid generation and solution procedure are separated with the discontinuities and special formulas are used to incorporate the jump conditions directly into the numerical model. This is the main idea behind this work considering moving boundary as a parameter.

To create a computational grid in body-fitted coordinates, two basic steps are required: (1) define an origin point, (2) specify the distribution (number and spacing) of grid nodes along the edges of the geometric regions. The automatic grid generator then takes over, and using an algebraic technique known as transfinite interpolation, creates a grid that simultaneously matches the edge node prescription and conforms to the irregular edges of the body-fitted geometry. Grid generation by algebraic methods produces high-quality numerical grids and allows for the very efficient integration of the thermal-flow field physics. Considering grid optimization, the designed grid optimization algorithm improves upon the transfinite interpolation method by carrying the grid generation process one step further. It uses automatically generated grid as an initial approximation to a higher quality grid system derived utilizing the technique of PDE grid generation. This technique offers advantages over purely algebraic methods:

- good control over the skewness and spacing of the derived grid on surface interiors, while simultaneously allowing complete control over the grid spacing (node distribution) on surface edges as well as moving boundary,
- an ability to produce unique, stable, and smooth grid distributions free of interior maxima or minima (inflection points) in body-fitted coordinates.

Parabolic grid generation works well with irregularly shaped geometries and can produce grids that are highly conformal with the edges of individual computational surfaces. The means for grid generation should not be dictated by the limitations of a given specific field solution procedure and conversely the method that determines the field should accept as input an arbitrary set of coordinate points which constitutes the grid. In general, of course, these two operations can never be totally independent because the logistic structure of the information, the location of outer boundaries, the nature of coordinate and the types of grid singularities are items that have to be coordinated closely between the field solver and the grid generator (Eriksson, 1982).

Grid generation for multidimensional geometries using transfinite interpolation functions was studied by Coons (1967), Cook (1974), Gordon and Hall (1973) and Ettouney and Brown (1983) successfully modeled slightly nonplanar interfaces by using an algebraic grid generation system where the interface was described in terms of univariate function.

Although grid generation is the core of most numerical algorithms for phase change problems or nonphase change problem, little effort has been reported on phase change problems, particularly the problem which couples the grid generation algorithm with the heat transport equations.

The present paper introduces the novel numerical approaches for melting problems which extend the range of initial condition and boundary condition in case of multiple constant temperature heat sources that can be covered. They will also permit a continuous determination of the multidimensional melting front and indicate the internal temperature distribution with a greater degree of boundary complexity and offers the highest overall accuracies and smooth grid point distribution. Numerically, for generating a boundary/interface fitted coordinate system, structured grids are initialized using transfinite interpolation algebraic techniques and the quality of structured grids can be significantly improved by applying parabolic-PDE methods. These methods iteratively solve unsteady conduction’s equation together with moving boundary condition during the melting process considering conduction as the only mode of heat transfer in both the unfrozen layer and the frozen layer.

2. Modeling formulation

The two-dimensional system illustrated schematically in Fig. 1 is considered. Initially, the walls are all insulated and the rectangular cavity is filled with a porous medium (PM) consisting of the glass beads and phase change material (PCM) in the solid state (ice), both at the fusion temperature \( T_f \). Multiple constant temperature \( (T_H) \) heat sources are located at the bottom wall. At time \( t = 0 \), the melting process upwardly begins. The applicable differential equations for two-dimensional heat flow with constant thermal properties for the unfrozen and frozen layers are, respectively,

\[
\frac{\partial T_l}{\partial t} = \alpha_l \left( \frac{\partial^2 T_l}{\partial x^2} + \frac{\partial^2 T_l}{\partial z^2} \right) + \left( \frac{\partial T_l}{\partial z} \right) \frac{dz}{dt},
\]

\[
\frac{\partial T_s}{\partial t} = \alpha_s \left( \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial z^2} \right) + \left( \frac{\partial T_s}{\partial z} \right) \frac{dz}{dt},
\]

where the last terms of Eqs. (1) and (2) result from a coordinate transformation attached to the moving boundary. In the unfrozen layer, internal natural convection can be neglected because the presence of glass beads minimizes the effect of natural convection current.

Eqs. (1) and (2) are based on the following assumptions:

1. the temperature field can be assumed to be two-dimensional;
2. the thermal equilibrium exists between PCM and PM; this is possible when the porous matrix has a little larger thermal conductivity than the PCM, and the interphase heat transfer can be properly neglected;
3. properties of PM are isotropic.

The boundary conditions of Eqs. (1) and (2) are:

(a) the localized heating condition at the bottom horizontal wall, where the multiple constant temperature \( (T_H) \) heat sources are applied:

\[ x_{H_1} \leq x_l \leq x_{H_2}; \quad T = T_H, \]

\[ x_{I_1} \leq x_c \leq x_{I_2}; \quad T = T_I, \]

\[ x_{F_1} \leq x_r \leq x_{F_2}; \quad T = T_{HF}; \]

(3)
(b) adiabatic condition: the walls except the position of localized heating condition are all insulated;

\[ \frac{\partial T}{\partial x} = \frac{\partial T}{\partial z} = 0; \]

(4)

(c) moving boundary condition;

the moving boundary condition (Stefan condition), which is obtained from a consideration of the energy balance at the interface between the unfrozen layer and frozen layer provides the following equation:

\[
\left( \lambda_s \frac{\partial T_s}{\partial z} - \lambda_l \frac{\partial T_l}{\partial z} \right) \left[ 1 + \left( \frac{\partial z_{\text{mov}}}{\partial x} \right)^2 \right] = \rho_s L_s \frac{\partial z_{\text{mov}}}{\partial t},
\]

(5)

where \( \partial z_{\text{mov}}/\partial t \) is the velocity of fusion front or melting front, and \( L_s \) the latent heat of fusion. To avoid changes in the physical dimensions as the melting front progresses, \( \rho_s / \rho_l \) will be specified. In this study, the thermal conductivity, \( \lambda_l \) and \( \lambda_s \) are bulk-average values for the glass beads and the water or ice, respectively.

3. Grid generation technique

Generally, two types of structured grid generation are currently in use. They are algebraic method or transfinite interpolation method and PDE method. Transfinite interpolation provides a relatively easy way of obtaining an initial grid that can be refined and smoothed by other techniques, whether algebraic, PDE method. For more complex geometries, such as in this work, it is preferable to construct the grid by transfinite interpolation initially, and to refine the grid filled in Cartesian coordinates in the interior of a domain by parabolic-PDE method subsequently.

3.1. Transfinite interpolation (TFI)

The method of constructing a two-dimensional boundary-conforming grid for a system is a direct algebraic approach based on the concept of TFI. In this method, no partial differential equations are solved to obtain the curvilinear coordinates, and the same system is used for the entire domain. The algebraic technique can be easier to construct than PDE methods, and gives also easier control over grid characteristics such as orthogonality and grid point spacing. However, this method is sometimes criticized for allowing discontinuities on the boundary to propagate into the interior and for not generating grids as smooth as those generated by PDE method.

The technique used for transfinite interpolation here is a significant extension of the original formulation by Gordon and Hall (1973). It is possible to initially generate global grid system with geometry specifications only on the outer boundaries of the computation domain and yet to obtain a high degree of local control.

Fig. 2 illustrates the present method of constructing a two-dimensional boundary-conforming grid for a system, which is a direct algebraic approach based on the concept of transfinite or multivariate interpolation. It is possible to initially generate global single plane transformations with geometry specifications only on outer boundaries of the computational domain.

Let \( f(u, w) = (x(u, w), z(u, w)) \) denote a vector-valued function of two parameters \( u, w \) defined on the region \( u_1 \leq u \leq u_{\text{max}}, \quad w_1 \leq w \leq w_{\text{max}} \). This function is not known throughout the region, only on certain planes (Fig. 2). The transfinite interpolation procedure then gives the interpolation function \( f(u, w) \) by the recursive algorithm:

\[
f^{(1)}(u, w) = A_1(u) \cdot f(1, w) + A_2(u) \cdot f(u_{\text{max}}, w),
\]

\[
f(u, w) = f^{(1)}(u, w) + B_1(w) \cdot [f(u, 1) - f^{(1)}(u, 1)] + B_2(w) \cdot [f(u, w_{\text{max}}) - f^{(1)}(u, w_{\text{max}})],
\]

(6)

where \( A_1(u), A_2(u), B_1(w) \) and \( B_2(w) \) are defined by the set of univariate blending functions, which only have to satisfy
the conditions:
\[ A_{1(1)} = 1, \quad A_{1(u_{\text{max}})} = 0, \]
\[ A_{2(1)} = 0, \quad A_{2(u_{\text{max}})} = 1, \]
\[ B_{1(1)} = 1, \quad B_{1(u_{\text{max}})} = 0, \]
\[ B_{2(1)} = 0, \quad B_{2(u_{\text{max}})} = 1. \] (7)

Further, the general form in algebraic equations can be defined as
\[ A_{1(u)} = \frac{u_{\text{max}} - u}{u_{\text{max}} - 1}, \quad A_{2(u)} = 1 - A_{1(u)}, \]
\[ B_{1(u)} = \frac{u_{\text{max}} - w}{u_{\text{max}} - 1}, \quad B_{2(u)} = 1 - B_{1(u)}. \] (8)

The grid motion defined from a moving boundary motion is modeled using a Stefan condition (Eq. (5)) with a transfinite mapping technique.

The boundary fitted grid generation mapping discussed in this section forms the basis for the interface tracking mapping. However, the mapping now must match the interface curve on the interior of physical domain in addition to fitting the outer physical boundary. In addition, the system must be adaptive since the grid lines must change to follow the deforming interface while maintaining as much smoothness and orthogonality as possible.

### 3.2. PDE method

In the proposed grid generation mapping, all grids discussed and displayed have been crouched in terms of finite difference formulation, with the understanding that whatever nonuniform grid exists in the physical space, there exists a transformation which will recast it as a uniform rectangular grid in the computational space. The finite difference calculations are then made over this uniform grid in the computational space, after which the field results are transferred directly back to the corresponding points in the physical space. The purpose of generating a smooth grid that conforms to physical boundaries of problem is, of course, to solve the partial differential equations specified in the problem by finite difference scheme, capable of handling general nonorthogonal curvilinear coordinates.

Fig. 1 shows that, as melting proceeds, the melting front denoted by \( z_{\text{mov}} \) is formed. Due to the existence of this melting front, the frozen and unfrozen domains are irregular and time dependent. To avoid this difficulty, a curvilinear system of coordinates is used to transform the physical domain into rectangular region for the computational domain.

It is convenient to introduce a general curvilinear coordinate system as follows (Anderson Jr., 1995):
\[ x = x(\xi, \eta), \quad z = z(\xi, \eta) \quad \text{or} \quad \xi = \xi(x, z), \quad \eta = \eta(x, z). \] (9)

The moving boundaries are immobilized in the dimensionless \((\xi, \eta)\) coordinate for all times. With the details omitted, the transformation of Eqs. (1), (2) and (5) can be written respectively as
\[
\frac{\partial T_1}{\partial t} = \frac{a_l}{J^2} \left( 2 \frac{\partial^2 T_1}{\partial \xi^2} - 2 \beta \frac{\partial^2 T_1}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 T_1}{\partial \eta^2} \right) + \frac{a_l}{J^3} \left[ \left( \frac{\partial^2 T_1}{\partial \xi^2} \right) \frac{\partial^2 T_1}{\partial \xi \partial \eta} - \left( \frac{\partial T_1}{\partial \eta} \right)^2 \right] - 2 \beta \frac{\partial^2 T_1}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 T_1}{\partial \eta^2} + \frac{1}{J} \left( x \frac{\partial T_1}{\partial \eta} \right) \frac{dz}{dt},
\]
\[
\frac{\partial T_2}{\partial t} = \frac{a_s}{J^2} \left( 2 \frac{\partial^2 T_2}{\partial \xi^2} - 2 \beta \frac{\partial^2 T_2}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 T_2}{\partial \eta^2} \right) + \frac{a_s}{J^3} \left[ \left( \frac{\partial^2 T_2}{\partial \xi^2} \right) \frac{\partial^2 T_2}{\partial \xi \partial \eta} - \left( \frac{\partial T_2}{\partial \eta} \right)^2 \right] + \frac{a_s}{J^3} \left[ \left( \frac{\partial^2 T_2}{\partial \xi^2} \right) \frac{\partial^2 T_2}{\partial \xi \partial \eta} - \left( \frac{\partial T_2}{\partial \eta} \right)^2 \right] + \frac{1}{J} \left( x \frac{\partial T_2}{\partial \eta} \right) \frac{dz}{dt},
\]
\[
\left\{ \lambda_s \frac{1}{J} \left( x \frac{\partial T_2}{\partial \eta} \right) - \lambda_l \frac{1}{J} \left( x \frac{\partial T_1}{\partial \eta} \right) \right\} \times \left\{ 1 + \left( \frac{1}{J} \left[ \frac{\partial z_{\text{mov}}}{\partial \xi} - \left( \frac{\partial z_{\text{mov}}}{\partial \eta} \right)^2 \right] \right) \right\} = \rho_s L_s \frac{\partial z_{\text{mov}}}{\partial t}.
\] (12)

where \( J = x^2 + z_{\eta} \cdot z_{\xi} - x_{\eta} \cdot z_{\xi}, \quad x = x^2 + z_{\eta}^2, \quad \beta = x_{\xi} \cdot x_{\eta} + z_{\xi} \cdot z_{\eta}, \quad \gamma = x^2 + z_{\eta}^2 + x_{\xi} \cdot x_{\eta} + x_{\xi} \cdot z_{\eta} \), \( z_{\xi} \) and \( z_{\eta} \) denote partial derivatives, \( J \) is the Jacobian, \( \beta, \alpha, \gamma \) are the geometric factors; and \( \eta, \xi \) are the transformed coordinates.

### 4. Solution method

It is known that the inherent difficulties in the conventional numerical methods (pure parabolic grid generators) for melting or freezing problems suggest the use of combined transfinite interpolation and PDE methods in most instances. Although conventional numerical methods can be used to obtain satisfactory results, there are problems of large time consumption and control functions that are often difficult to determine. Therefore, the new method presented in this paper is generally preferable because it offers the highest overall accuracies and smooth grid point distribution. In addition, the boundary point discontinuities are smoothed out in the interior domain and orthogonality at boundaries can be maintained.
Table 1
Thermal property of the unfrozen layer and frozen layer

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unfrozen layer</th>
<th>Frozen layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ (kg/m³)</td>
<td>1942.0</td>
<td>1910.0</td>
</tr>
<tr>
<td>a (m²/s)</td>
<td>0.210 × 10⁻⁶</td>
<td>0.605 × 10⁻⁶</td>
</tr>
<tr>
<td>λ (W/mK)</td>
<td>0.855</td>
<td>1.480</td>
</tr>
<tr>
<td>C_p (J/kg K)</td>
<td>2.099 × 10³</td>
<td>1.281 × 10³</td>
</tr>
</tbody>
</table>

In this study, in order to initiate numerical simulation, a very thin layer of melt with a constant thickness was assumed to be present. This initial condition is obtained from the Stefan solution in the melt and a linear temperature distribution in the frozen layer. Tests revealed that the influence of \( z_{melt}(0) \) could be neglected as \( z_{melt}(0) \) was sufficiently small. The transient heat equations (Eqs. (10) and (11)) and the Stefan condition (Eq. (12)) are solved by using finite difference method using parameter values obtained from Table 1. A system of nonlinear equations results whereby each equation for the internal nodes can be cast into a numerical discretization.

Transient heat equation for unfrozen layer:

\[
T_{n+1}^{i}(k, i) = \left( \frac{1}{1+(2a_t \Delta t/J^2(k,i))((x(k, i)\Delta \zeta \Delta \eta) + (\gamma(k, i)\Delta \eta \Delta \eta))} \right) \times \left( T_n^{i}(k, i) + \frac{a_t \Delta t}{J^2(k,i)} \right) \\
+ \left( z(k, i) X(k, i + 1) + Z(k, i) \right) \Delta \zeta \\
+ \left( \frac{Z(k+1, i) - Z(k, i - 1)}{2 \Delta \eta} \right) \\
- \left( \frac{T_{i+1}^{n+1}(k+1, i) - T_{i}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \\
- 2\beta(k, i) \left( \frac{Z(k+1, i+1) - Z(k, i + 1)}{2 \Delta \eta} \right) \\
- \left( \frac{Z(k+1, i-1) - Z(k, i - 1)}{2 \Delta \eta} \right) \right) \right) / 2\Delta \zeta \\
+ \gamma(k, i) \left( \frac{T_{i+1}^{n+1}(k+1, i) + T_{i}^{n+1}(k, i - 1)}{\Delta \eta \Delta \eta} \right) + \frac{a_t \Delta t}{J^2(k,i)} \times \left( \frac{X(k, i) X(k, i + 1) + X(k, i, i - 1)}{\Delta \zeta} \right) \\
+ \left( \frac{Z(k+1, i) - Z(k, i - 1)}{2 \Delta \eta} \right) \\
- \left( \frac{T_{i+1}^{n+1}(k+1, i) - T_{i}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \\
- \left( \frac{Z(k+1, i+1) - Z(k, i + 1)}{2 \Delta \eta} \right) \\
+ \gamma(k, i) \left( \frac{T_{i+1}^{n+1}(k+1, i+1) + T_{i}^{n+1}(k, i - 1)}{\Delta \eta \Delta \eta} \right) \right) \right) .
\]

Transient heat equation for frozen layer:

\[
T_{s}^{n+1}(k, i) = \frac{1}{1+(2a_t \Delta t/J^2(k,i))((x(k, i)\Delta \zeta \Delta \eta) + (\gamma(k, i)\Delta \eta \Delta \eta))} \times \left( T_n^{s}(k, i) + \frac{a_t \Delta t}{J^2(k,i)} \right) \\
+ \left( \frac{X(k, i) X(k, i + 1) + X(k, i, i - 1)}{\Delta \zeta} \right) \\
+ \left( \frac{Z(k+1, i) - Z(k, i - 1)}{2 \Delta \eta} \right) \\
- \left( \frac{T_{s}^{n+1}(k+1, i) - T_{s}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \\
- \left( \frac{T_{i+1}^{n+1}(k+1, i) - T_{i}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \\
- \left( \frac{Z(k+1, i+1) - Z(k, i + 1)}{2 \Delta \eta} \right) \\
+ \gamma(k, i) \left( \frac{T_{i+1}^{n+1}(k+1, i+1) + T_{i}^{n+1}(k, i - 1)}{\Delta \eta \Delta \eta} \right) + \frac{a_t \Delta t}{J^2(k,i)} \times \left( \frac{X(k, i) X(k, i + 1) + X(k, i, i - 1)}{\Delta \zeta} \right) \\
+ \left( \frac{Z(k+1, i) - Z(k, i - 1)}{2 \Delta \eta} \right) \\
- \left( \frac{T_{s}^{n+1}(k+1, i) - T_{s}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \\
- \left( \frac{T_{i+1}^{n+1}(k+1, i) - T_{i}^{n+1}(k, i - 1)}{2 \Delta \zeta} \right) \right) .
\]
Stefan condition:

\[ Z^{n+1}(k, i) = Z^n(k, i) + \frac{\Delta t}{\rho_s L_s} \times \left( -\frac{\dot{\lambda}_s}{J(k-1, i)} \times \left( \frac{X(k-1, i+1) - X(k-1, i-1)}{2\Delta \zeta} \right) \right. \]

\[ \left. \times \left( \frac{3T_s(k, i) - 4T_l(k-1, i) + T_l(k-2, i)}{2\Delta \eta} \right) + \frac{\dot{\lambda}_s}{J(k+1, i)} \times \left( \frac{X(k+1, i+1) - X(k+1, i-1)}{2\Delta \zeta} \right) \right. \]

\[ \left. \times \left( \frac{-3T_s(k, i) + 4T_s(k+1, i) - T_s(k+2, i)}{2\Delta \eta} \right) \right) \]

\[ \times \left( \frac{Z^n(k+1, i) - Z^n(k-1, i)}{2\Delta \eta} \right) \]

\[ \times \left( \frac{Z^n(k+1, i) - Z^n(k-1, i)}{2\Delta \zeta} \right) \]

\[ \left. \times \left( \frac{Z^n(k, i+1) - Z^n(k, i-1)}{2\Delta \zeta} \right) \right) \]

\[ \left. \times \left( \frac{Z^n(k, i+1) - Z^n(k, i-1)}{2\Delta \eta} \right) \right) \}

The details of computational schemes and strategy for solving the combined transfinite interpolation functions (Eqs. (6)–(8)) and PDE (Eqs. (13)–(15)) are illustrated in Fig. 3.

5. Results and discussion

Numerical results are obtained for phase change in a rectangular cavity filled with a porous medium. The calculations are performed under the following conditions:

1. The time step of \( dt = 0.1 \) (s) is used for the computation of temperature field and location of melting front.
2. The number of cells is \( N = 120 \) (width) \( \times 100 \) (depth).
3. Iterations are carried out until relative error of \( 10^{-8} \) are reached.

In order to verify the accuracy of the present numerical algorithm, it is validated by performing simulations for a planar melting front in a pure ice slab. Initially, the temperature of 0°C is assigned throughout each layer. Thereafter, the constant temperature heat source \( (T_H = 100°C) \) is imposed on the bottom wall. The calculated front location is based on the thermal properties of ice and water. The results are then compared with analytical solution for the melting of a pure ice slab at the same condition. Fig. 4 clearly shows a good agreement between simulated and analytical solutions. Therefore, the present method can yield accurate solutions.

Fig. 5 shows the measured and simulated results of the melting front during melting of ice in a rectangular cavity filled with a porous medium. In this comparison, the single constant temperature heat source, \( T_H = 100°C \), is applied. The observation of the melting front depicted from the figure reveals that the simulated results and experimental results are qualitatively consistent. However, the experimental data are significantly lower than the simulated results. The spreading of the melt in the \( x \)-direction from experimental results is clearly shown. Discrepancy may be attributed to heat loss and nonuniform heating effect along the surface of supplied load. Numerically, the discrepancy may be attributed to uncertainties in the thermal and
physical properties data. In addition, the source of the discrepancy may be attributed to natural convection effect in liquid.

5.1. A melting front tracking grid generation system

The purpose of this subsection is to illustrate the efficiency of the grid generation system during the melting of ice in a rectangular cavity filled with a porous medium (porosity, $\phi = 0.38$) subjected to multiple constant temperature heat sources. Fig. 6(a) shows the initial reference grid for the domain generated by pure transfinite interpolation method. Figs. 6(b)–(g) show grids that fit curves that are typical of shapes seen during deformation of an interface with respect to elapsed times. The calculated front locations correspond the initial temperature of $0^\circ$C and applied boundary condition ($T_H = 90^\circ$C) given by Eq. (3). It can be seen how melting fronts progress with respect to elapsed times. During the initial stages of melting the shape of the interface in each region becomes flatter as the melting front moves further away from the fixed boundaries indicating principally one-dimensional heat flow. At later times, the curve on the interface gradually flattens indicating the two-dimensional effect.

In all figures, it is found that the grid is able to maintain a significant amount of orthogonality and smoothness both within the interior and along the boundary as the grid points redistribute themselves to follow the interface. These results show the efficiency of the present method for the moving boundary problem.

5.2. Melting process

The present work is to couple the grid generation algorithm with the transport equations. The thermal analysis during melting process will be discussed in this subsection. The simulations of temperature distribution within rectangular cavity filled with porous media in the vertical plane ($x - z$) corresponding to grid simulating the deformation of an interface (Figs. 6(a)–(g)) are shown in Figs. 7(a)–(g). When multiple constant temperature heat sources are applied, heat is conducted from the hotter region in unfrozen layer to the cooler region in frozen layer. At the initial stages of melting, the melting fronts remain square in shape indicating principally one-dimensional heat flow as explained in the previous subsection. Later, the melting fronts gradually exhibit a shape typical for two-dimensional heat conduction dominated melting. As the melting process persists, the melting rate progresses slowly. This is because most of heat conduction takes place the leading edge of unfrozen layer (melt layer) which is located further from frozen layer. Consequently, small amount of heat can conduct to the frozen layer due to the melt layer acting as an insulator and causing a slowly melting fronts to move with respect to elapsed times. It is observed that as the melting progresses, the melt layers at the leading edge expand wider but it expands less at the position further away from the applied boundary condition surface due to the phenomenon of heat transport as explained above.

It is observed that each hot region of the rectangular cavity shows signs of melting, while the outer edges display no obvious sign of melting indicating that the temperature does not exceed $0^\circ$C. Nevertheless, the leading edge of applied boundary condition surface displays sign of melting continuously.

This study shows the capability of the present method to correctly handle the phase change problem. With further quantitative validation of the present method, this method can be used as a tool for investigating in detail this particular melting of phase change slab at a fundamental level.

6. Conclusions

Mesh quality has the largest impact on solution quality. A high-quality mesh increases the accuracy of the computational thermal flow solution and improves convergence. Therefore, it is important to provide tools for obtaining and improving a mesh.

In this study, melting of ice in a rectangular cavity filled with a porous medium subjected to multiple constant temperature
Fig. 6. Grid simulating the deformation of an interface: (a) the initial reference grid for the domain (generated by pure transfinite interpolation method), (b) melting time of 60 s, (c) melting time of 120 s, (d) melting time of 180 s, (e) melting time of 240 s, (f) melting time of 300 s, and (g) melting time of 360 s.
Fig. 7. The simulations of temperature distribution (unit: °C) within rectangular phase change slab: (a) the initial stage of melting, (b) melting time of 60 s, (c) melting time of 120 s, (d) melting time of 180 s, (e) melting time of 240 s, (f) melting time of 300 s, and (g) melting time of 360 s.
heat sources has been investigated numerically. A generalized mathematical model and an effective calculation procedure are proposed. A preliminary case study indicates the successful implementation of the numerical procedure. A two-dimensional melting model is then validated against available analytical solutions and experimental results and subsequently used as a tool for efficient computational prototyping. Simulated results are in good agreement with available analytical solution and experimental results. The successful comparison with analytical solution and experiments should give confidence in the proposed mathematical treatment, and encourage the acceptance of this method as useful tool for exploring practical problems.

The next phase, which has already begun, is to couple the grid generation algorithm with the complete transport equations that determine the moving boundary front and buoyancy-driven convection in the liquid. The influence of adjusted meshes number in each layer on thermal flow solution will be investigated. Moreover, some experimental studies will be performed to validate numerical results.

Notation

\begin{align*}
a & \quad \text{thermal diffusivity, m}^2/\text{s} \\
C_p & \quad \text{specific heat capacity, J/kg K} \\
L & \quad \text{latent heat, J/kg} \\
t & \quad \text{time, s} \\
T & \quad \text{temperature, C} \\
x, z & \quad \text{Cartesian coordinates}
\end{align*}

Greek letters

\begin{align*}
\lambda & \quad \text{effective thermal conductivity, W/mK} \\
\phi & \quad \text{porosity}
\end{align*}

Subscripts

\begin{align*}
f & \quad \text{fusion} \\
i & \quad \text{initial} \\
j & \quad \text{layer number} \\
l & \quad \text{unfrozen} \\
s & \quad \text{frozen}
\end{align*}

Acknowledgment

The author is pleased to acknowledge Thailand Research Fund (TRF) for supporting this research work.

References


