ments are readily available in the market. One of these nonlinear elements, the voltage-dependent resistor may be connected to natural convection and thermal radiation modes. The best-known family of software tools for electric circuit analysis that include linear and nonlinear elements is named Simulation Program with Integrated Circuit Emphasis (SPICE) (Nagel [7]). In order to use SPICE, the analyst must first provide a complete description of the electric circuit. This step is done through a series of element statements, with one such statement for every element in the circuit. Second, these statements are followed by control statements, which instruct the program what needs to be calculated.

Conclusions

Under the premises of the powerful lumped model, the RC electric circuit that simulates forced convection cooling (linear heat transfer mode) with a constant convection coefficient has been a staple in the theory of heat conduction for many decades [3]. However, for the equally important situations of cooling by natural convection (nonlinear heat transfer mode) or thermal radiation (nonlinear heat transfer mode) the appropriate electric circuits do not exist in the heat transfer literature. These shortcomings were precisely the source of motivation for undertaking this study on advanced electro-thermal analogies.

When the cooling occurs by natural convection, the controlling natural convection coefficient is susceptible to the shape of the solid body and its orientation. Because of this, the Nu correlation equations may involve one, two or even three terms and the appropriate electric circuits may vary slightly from the one sketched in Fig. 2. In contrast, the electric circuit for thermal radiation cooling regardless of the body shape is given by Fig. 3.

Nomenclature

a, b = constants in Eq. (10)
C = thermal capacitance, J/K
C_e = electric capacitance, farads
e = constant in Eq. (14)
$E - E_\infty$ = electric potential, volts
h = forced or natural convection coefficient, W/m^2·K
$h_r(T)$ = radiation coefficient, W/m^2·K
j = current flow, amperes
Q = heat flow, W
R = thermal resistance, K/W
R_e = electric resistance, ohms
$T - T_\infty$ = thermal potential, K

Greek letters

θ = temperature transformation for natural convection, $T - T_\infty$, K

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Experimental Validation of a Combined Electromagnetic and Thermal Model for a Microwave Heating of Multi-Layered Materials Using a Rectangular Wave Guide

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The heating of multi-layered materials by microwave heating with rectangular wave guide has been investigated numerically and experimentally. The multi-layered materials, which consist of the layer of higher dielectric material (antireflection layer) and lower dielectric material (sample), have the convergent effect of the incident microwave in sample, and it can change the heating pattern in the sample with ease. In this study, the effect of an antireflection layer thickness on the heating process is clarified in detail, considering the interference between incidents and reflected waves in the dielectric materials. Based on a model combining the Maxwell and heat transport equations, the results showed that when a layer of lower dielectric material is attached in front of sample, the microwave energy absorbed and distribution of temperature within the sample are enhanced. The predicted results are in agreement with experimental results for microwave heating of multi-layered materials using a rectangular wave guide.

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Keywords: Heat Transfer, Materials, Microwave, Modeling, Numerical Methods, Thermal

1 Introduction

The microwave heating of layered materials has been studied by many investigators, including Etternberg [1], Gori et al. [2], Nikawa et al. [3], Ayappa et al. [4] and Lin and Ghandhi [5]. Although most previous investigations are deplete with numerical simulations of simple model in one-dimensional form, there are only few papers have been reported on microwave heating of layered materials inside a rectangular wave guide, especially a full comparison of the space-time evolution of temperature between simulated results with experimental heating data. Part of the reason may be that analysis of microwave heating of layered materials is considerably more challenging due to the influence of dielectric properties in each layer, resulting to the complex interactions of microwave field with layered materials.

For microwave heating process, deep and localized heating technique for the dielectric material layer are sometimes demanded. When microwave fields are used, the depth of penetration is generally shallow and it is difficult to heat deep-lying layer and relatively large layered volumes without excessive surface heating. In order to overcome this difficulty, using the multi-layered material approach is one of the best means. The authors

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suggest that the microwave energy absorbed and distribution of temperature within the sample are enhanced when a layer of lower dielectric material (called antireflection layer) is attached in front of sample (called higher dielectric material).

The characteristics of microwave heating of multi-layered materials studied here are potentially applicable to design of electromagnetic hyperthermia system for the treatment of cancer. The using of lower dielectric material as a mean of antireflection layer allowing more of the microwave energy can deposit deep inside the human tissues to keep them at the desired temperature elevation. Furthermore, the concept of microwave heating of multi-layered materials can be also useful for explaining the drying phenomenon in a fundamental level, particularly the complex interactions of microwave field with multi-layered materials. Considering microwave drying of porous materials (Ratanadecho et al. [6]), the drying layer takes place on a front retreating from the surface into the interior of the sample dividing it into two layers, dry and wet layers. Inside the drying front, the sample is wet, i.e., the voids contain liquid water and this layer acts as higher dielectric material. Outside the drying front, no liquid water exists, all water is in vapor state and the dry layer acts as the lower dielectric material. The changing of dry layer thickness (or lower dielectric material) would change the intensity of electric field, wavelength and location of maximum microwave energy absorbed with respect to drying times. This phenomenon explains why the understanding of interactions of microwave field with layered materials much be carefully performed.

Since the microwave heating of layered material is very complicated, consequently, the study in microwave heating of layered materials should be systematically studied. This work, the microwave heating of multi-layered material based on a two-dimensional model with experimental data, in which the microwave of TE_{10} mode operating at a frequency of 2.45 GHz, is completely presented in order to validate the possibility for using the multi-layered materials as a heating sample. The result presented here provides a basis for fundamental understanding of microwave heating of multi-layered materials.

2 Experimental Configuration

Figure 1 shows the experimental apparatus. The microwave system was a monochromatic wave of TE_{10} mode operating at a frequency of 2.45 GHz. Microwave energy was generated by magnetron (Micro Denshi Co., model UM-1500), it was transmitted along the *z*-direction of the rectangular wave guide with inside dimensions of 110 mm×54.61 mm toward a water load that was situated at the end of the wave guide. The water load (lower absorbing boundary) ensured that only a minimal amount of microwave was reflected back to the sample. The sample heated was a packed bed of 50 mm in thickness, which was composed of glass beads and water, called a higher dielectric material. The antireflection layer was also a packed bed ($d=1.0$ mm) which was composed of glass beads ($d=1.0$ mm) and air, called a lower dielectric material. The sample and the antireflection layer are arranged in series perpendicular to direction of irradiation via a rectangular wave guide. During the experiment, output of magnetron was adjusted at 1000 W. The powers of incident, reflected and transmitted waves were measured by a wattmeter using a directional coupler (Micro Denshi Co., model DR-5000). The temperature was measured with a Luxtron fluroptic thermometer model 790 (accurate to ±0.5°C).

3 Analysis of Microwave Heating Using a Rectangular Wave Guide

Physical Model. Figure 2 shows the physical model for the microwave heating of multi-layered materials using rectangular wave guide. Since microwave of TE_{10} mode which propagates in rectangular wave guide is uniformed in *y*-direction, the electromagnetic field can be considered in two-dimensional model on *x*-*z* plane. Corresponding to electromagnetic field, temperature fields also can be considered in two-dimensional model. The model proposed is based on the following assumptions:

1) The absorption of microwave by air in rectangular wave guide is negligible.
2) The walls of rectangular wave guide are perfect conductors.
3) All materials are non-magnetic.
4) The packed bed is an isotropic medium and thermal properties of packed bed are constant.
5) The liquid phase is incompressible fluid.
6) The effect of the sample container on the electromagnetic, velocity and temperature fields can be neglected.

Maxwell's Equation. Assuming the microwave of TE_{10} mode, the governing equations for the electromagnetic field can be written in term of the component notations of electric and magnetic field intensities (Ratanadecho et al. [6,7])

$$\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t} \tag{1}$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \tag{2}$$

$$-\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = \sigma E_y + \varepsilon \frac{\partial E_y}{\partial t} \tag{3}$$

where, permittivity or dielectric constant ε, magnetic permeability μ and electric conductivity σ are given by

$$\varepsilon = \varepsilon_0 \varepsilon_r, \quad \mu = \mu_0, \quad \sigma = 2\pi f \varepsilon \tan\delta \tag{4}$$

The dielectric properties are assumed to vary with temperature during the heating process. To determine the functional dependence of the temperature [8], the theory of mixing formulas is used (Wang and Schmugge [9]). Further, all boundary conditions for solving Maxwell's equations have been already presented in previous work (Ratanadecho et al. [6]).

Heat Transport Equation. The temperature of the material exposed to incident wave is obtained by solving the conventional heat transport equation with the microwave power included as a local electromagnetic heat generation term:

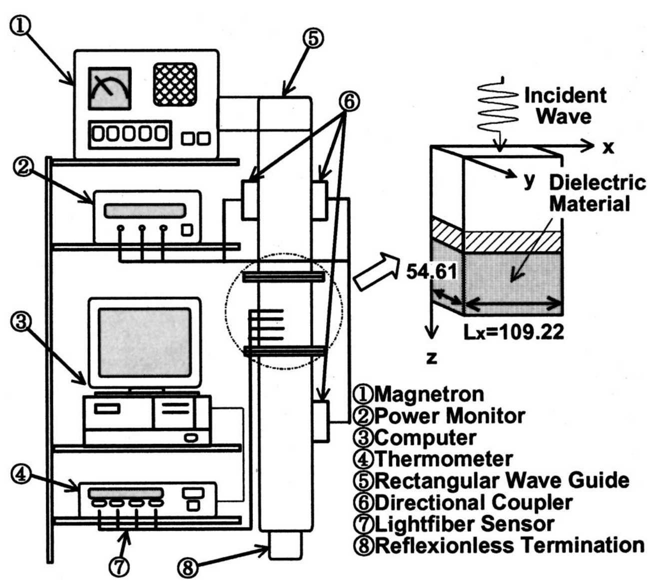

Fig. 1 Schematic of experimental facility

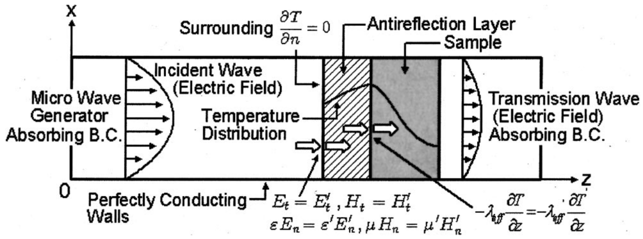

Fig. 2 Physical model

$$\frac{\partial T_j}{\partial t}=a_j\left(\frac{\partial^2 T_j}{\partial x^2}+\frac{\partial^2 T_j}{\partial z^2}\right)+\frac{Q_j}{\rho_j\cdot C_{pj}} \tag{5}$$

where, T is temperature, a is thermal diffusivity, ρ is density, c_p is heat capacity at constant pressure, and subscript j denotes lower or higher dielectric materials. The local electromagnetic heat generation term Q_j is directly depended upon the electric field distribution defined as:

$$Q_j=2\pi\cdot f\cdot\varepsilon_0\cdot\varepsilon_r(\tan\delta)E_y^2 \tag{6}$$

The boundary conditions for solving heat transport equation are shown in Fig. 2.

4 Results and Discussion

The experimental results for the microwave heating of multi-layered materials were compared with predictions from mathematical model. Some of the input data for electromagnetic and thermo-physical properties are given in Table 1.

In order to predict the electromagnetic and temperature fields, a finite difference time domain (FDTD) method is employed. The total 110×250 cells in computational domain are used in the numerical calculation. Since the propagating velocity of microwave is very fast compared with the rate of heat transfer, different time steps of $dt=1$ [ps] and 1 [s] are used for the computation of the electromagnetic field and temperature profile and the spatial step size of $dx=dz=1$ [mm] is selected.

4.1 The Variation of the Reflection Rate. Figure 3 shows the variation of the reflection rate versus the thickness of antireflection layer. It is found that the reflection rate strongly depends on the thickness of antireflection layer and displays a wavy behavior with respect to the thickness. The maximum and minimum reflection rates that occur in each thickness of antireflection can be clearly seen in that figure. This variation of the reflection rate is caused due to the interference between transmitted and reflected waves in an antireflection layer. The predicted results are in good agreement with the experimental results.

4.2 The Distribution of Electric Field. To understand the distribution of electrical field inside the rectangular wave guide and the multi-layered materials, the simulation analysis is required. For different heating sample configuration, the electric field pattern even with the same microwave power level. In Figs. 4–6, the simulation of the typical electric field of TE_{10} mode

Table 1 The electromagnetic and thermo-physical properties used in computations

$\varepsilon_0=8.85419\times10^{-12}$[F/m],	$\mu_0=4.0\pi\times10^{-7}$[H/m]	
$\varepsilon_{ra}=1.0$,	$\varepsilon_{rp}=5.1$	
$\mu_{ra}=1.0$,	$\mu_{rp}=1.0$,	$\mu_{rw}=1.0$
$\tan\delta_a=0.0$,	$\tan\delta_p=0.01$	
$\rho_a=1.205$[kg/m^3],	$\rho_p=2500.0$[kg/m^3],	$\rho_w=1000.0$[kg/m^3]
$C_{pa}=1.007$[kJ/(kg·K)],	$C_{pp}=0.80$[kJ/(kg·K)],	$C_{pw}=4.186$[kJ/(kg·K)]
$\lambda_a=0.0262$[W/(m·K)],	$\lambda_p=1.0$[W/(m·K)],	$\lambda_w=0.610$[W/(m·K)]

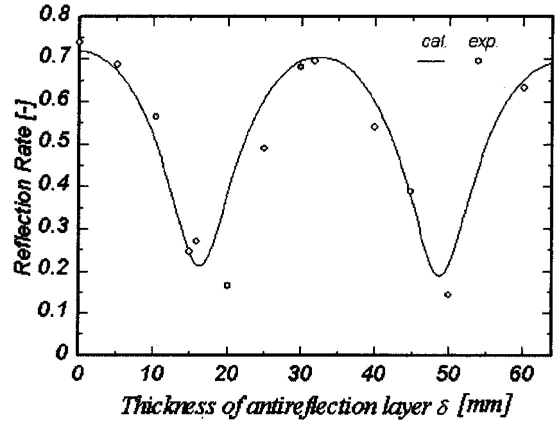

Fig. 3 Variation of the reflection rate versus thickness of antireflection layer

along the center axis ($x=54.61$) of rectangular wave guide, for the cases of without and with antireflection layer, is presented. The vertical axis represents the intensity of the electric field E_y, which is normalized to the amplitude of the input electromagnetic wave, E_{yin}.

In Fig. 4, corresponding to that case without antireflection layer (δ=0 mm), since the microwave passing through cavity having low permittivity is directly irradiated to the sample having high permittivity, large part of microwaves are reflected from the surface of the sample (as referred to Fig. 3) and a stronger standing wave with larger amplitude is formed within the cavity by the superimposition of forward wave propagated from the wave input region and of the reflected wave from the surface of the sample.

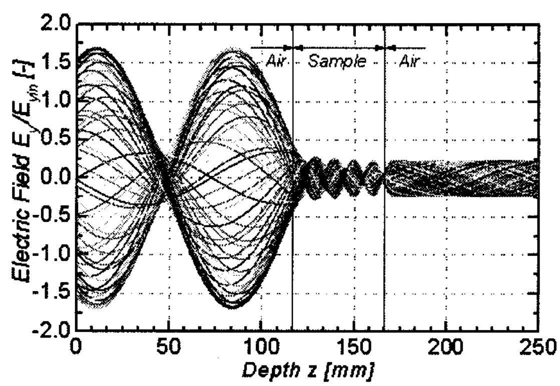

Fig. 4 Distribution of electric field in case without antireflection layer on the sample

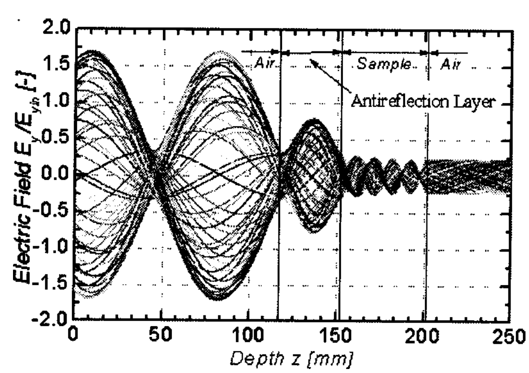

Fig. 5 Distribution of electric field in the case that antireflection layer is attached on the sample (δ=32 mm)

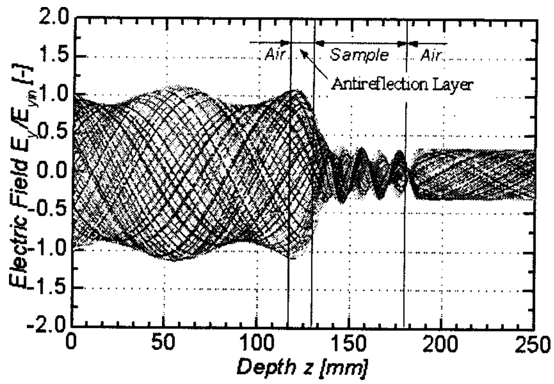

Fig. 6 Distribution of electric field in case that antireflection layer is attached on the sample (δ=16 mm)

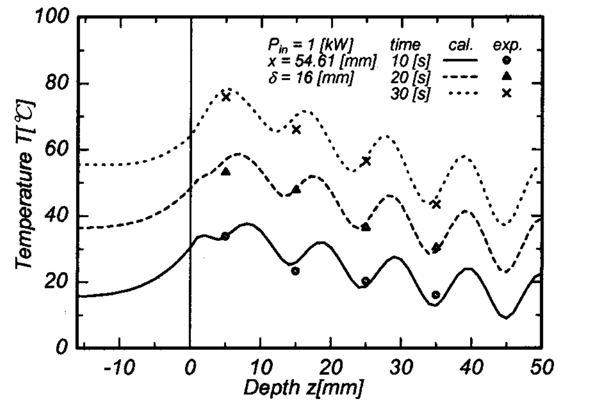

Fig. 8 Temperature distributions in case that antireflection layer (δ=16 mm) is attached on the sample

However, the electric field within the sample is almost extinguished where the electric field attenuates owing to microwave energy absorbed, and thereafter the microwave energy absorbed is converted to the thermal energy.

Figure 5 shows the distribution of electric field for the case of antireflection layer with thickness of 32 mm is attached in front of sample. It can be seen that the distribution of electric field for this case is nearly the same as that without antireflection layer (Fig. 4). Therefore, the presence of antireflection layer at thickness of 32 mm slightly affects the distribution of electric field, due to the large part of microwaves are reflected from the surface (as referred to Fig. 3).

Figure 6 shows the distribution of electric field for the case of antireflection layer with thickness of 16 mm is attached in front of sample. With the presence of antireflection layer, the small part of microwave is reflected from the surface of the sample (as referred to Fig. 3). Since the large part of microwaves can further penetrate into the sample, the wave reflected from the sample-air interface at the lower surface has the same order as propagating wave. The propagating and reflected waves will contribute to the standing wave pattern with the larger amplitude and wavelength inside the sample, such pattern can lead to higher microwave energy absorbed in the interior in comparison with other cases.

4.3 The Distribution of Temperature Field. The predictions from mathematical model are compared with experimental microwave heating data in Figs. 7 and 8, which corresponds to that of initial temperature of 0°C at heating times of 30 s, along with the center axis (x = 54.61 mm) of rectangular wave guide. It is clearly seen that the distribution of temperature in the sample is wavy behavior corresponding to that of electromagnetic field and the peaks of the temperature distribution decay slowly along the propagation direction. The distribution of temperature for case of antireflection layer with thickness of 32 mm is nearly the same as that without antireflection layer (Fig. 7), except at the surface part of the sample where the temperature drops due to the conduction of heat to the antireflection layer.

When the antireflection layer with thickness of 16 mm is attached in front of sample, the distribution of temperature is higher than those cases without antireflection layer and attaching the antireflection layer with thickness of 32 mm. This implies that with the presence of suitable antireflection thickness, a stronger standing wave with larger amplitude will be formed within the sample, consequently, a higher maximum temperature will be produced. Compared to the results for other cases (δ=0 mm and δ=32) at

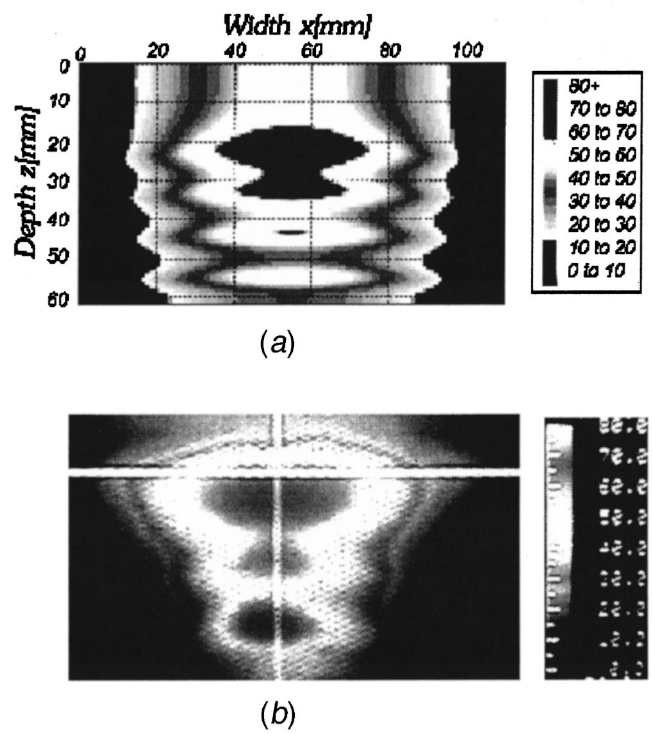

Fig. 9 Comparison between simulated results (a) and experimental results (b) for microwave heating in the case that antireflection layer is attached on the sample (δ=16 mm., t=30 s): (*a*) simulated temperature distributions (Units: °C); and (*b*) measured temperature distributions (Units: °C).

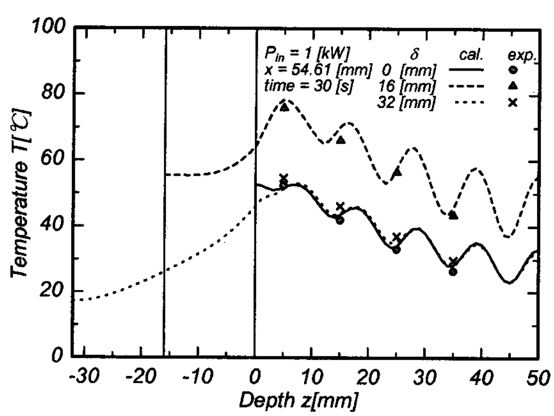

Fig. 7 Temperature distributions as a parameter of thickness of antireflection layer

heating times of 30 s, the maximum temperature is about 78°C. For other cases (δ=0 mm and δ=32) with the same microwave power level at the same heating time, it is about 50°C, which is about 56 percent higher. The distribution of temperature with respect to times for the case of antireflection layer with thickness of 16 mm is also shown in Fig. 8. The wavy behavior of temperature distributions appear and rapidly raise with elapsed time because of a stronger standing wave with larger amplitude is formed in the sample and the antireflection layer protects the reflected wave from the surface. The predicted results are in agreement with experimental results for microwave heating of multi-layered materials using a rectangular wave guide.

Furthermore, the distribution of temperature within the multi-layered materials in the vertical plane (x-z) is shown in Fig. 9. The result shows the greatest temperature at the center of heating sample where the microwave energy absorbed is maximum. It can be seen that the agreement between the two heating patterns is good, particularly concerning the location of the hot region.

From this study, the result predicts the possibility of effective depth and localized heating in a dissipative medium such as dielectric material. It seems that a proper selection of the suitable antireflection layer thickness could lead to the increase in the penetration depth and the heat generation in the dielectric materials.

5 Conclusions

The experimental and numerical of two-dimensional model for microwave heating of multi-layered materials is presented in order to validate the possibility for using the multi-layered materials as a heating sample. In this study, the influence of the thickness of antireflection layer on the microwave energy absorbed and heating pattern that develop within multi-layered materials is clarified in detail. The results showed that the microwave energy absorbed as well as wavelength and distribution of temperature within the sample are enhanced when a suitable thickness of antireflection layer (lower dielectric material) is attached in front of sample (higher dielectric material). This is due to attaching the suitable thickness of antireflection layer decreases the reflected wave from the surface of sample (higher dielectric material), the latter arises from the fact that the large parts of microwaves can penetrate further into the sample. The propagating and reflected waves at each interface will contribute to the stronger standing wave with the larger amplitude and wavelength, resulting in a higher microwave energy absorbed in the interior in comparison with other cases. On the other hand, in other cases that the reflected wave is still higher resulting in a lower microwave energy absorbed in the interior. The predicted results are in agreement with experimental results for microwave heating of multi-layered materials using a rectangular wave guide.

Nomenclature

a = thermal diffusivity [m²/s]
E = electric field intensity [V/m]
f = frequency of incident wave [Hz]
H = magnetic field intensity [A/m]
P = power [W]
Q = electromagnetic heat generation [W/m³]
$\tan\delta$ = dielectric loss tangent coefficient
t = time [s]

Greek letters

δ = layer thickness [m]
ε = permittivity or dielectric constant [F/m]
ε_r = relative permittivity or relative dielectric constant
λ = thermal conductivity of materials [W/m²K]
μ = magnetic permeability [H/m]
σ = electric conductivity [S/m]
ω = angular frequency [rad/s]

Subscripts

0 = free space
a = air
in = input
p = particle
r = relative
w = water
x,y,z = coordinates

Superscripts

$'$ = interfacial position

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