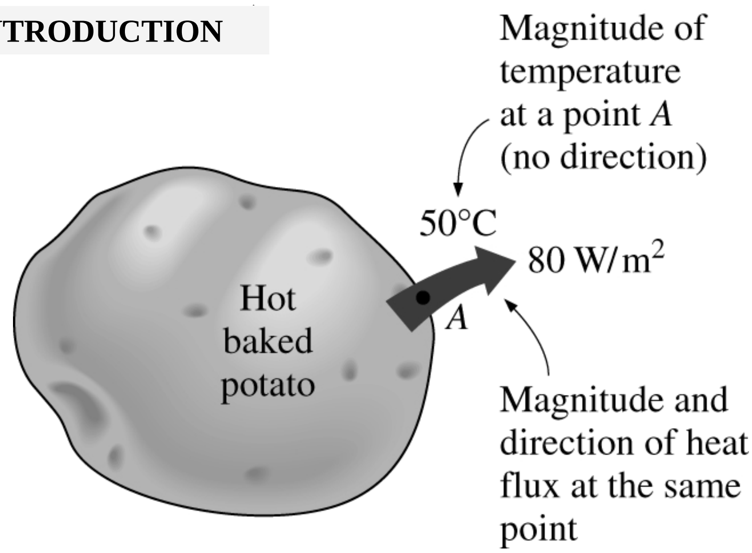
FIGURE 2–1

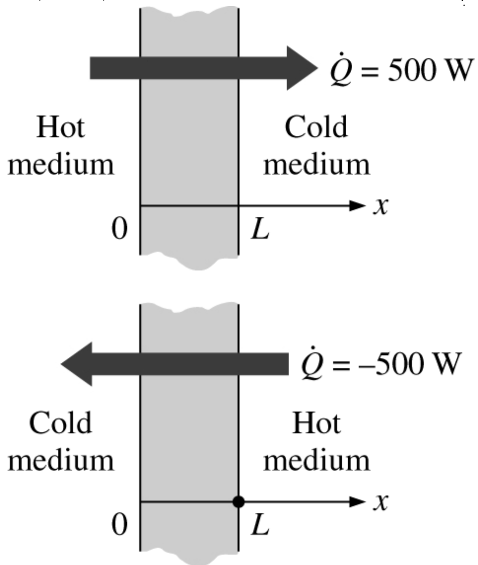
## 2.1 INTRODUCTION




1



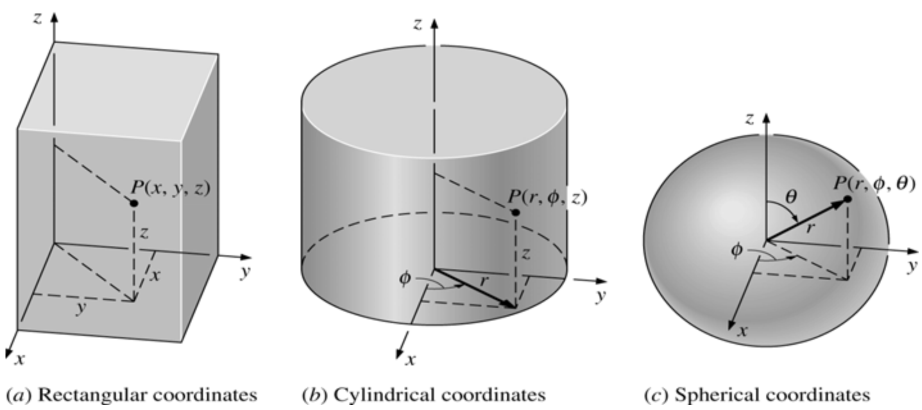
FIGURE 2–2




Indicating direction for heat transfer (positive in the positivedirection;negative in the negative direction)

2

## FIGURE 2–3

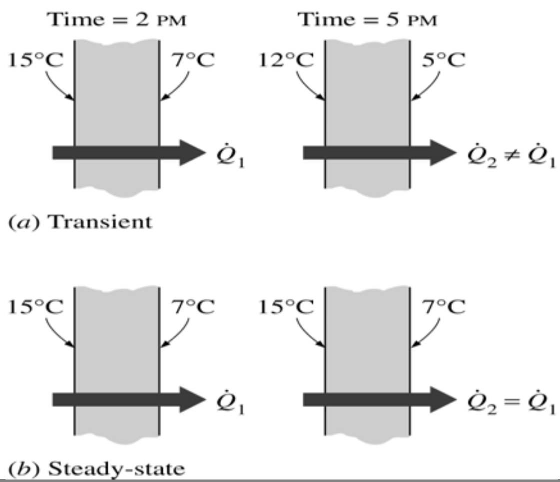



The various distances and angles involved when describing the location of a point in different coordinate systems




## FIGURE 2–4

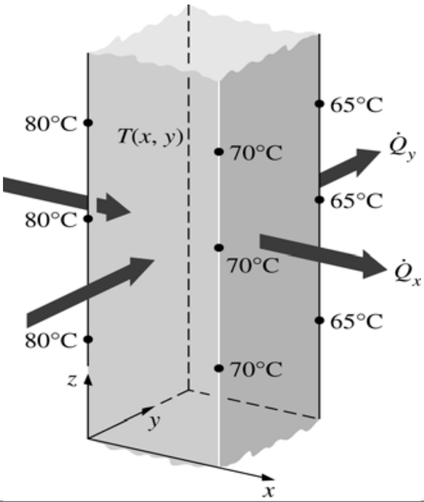
**Steady versus Transient Heat Transfer**




**Steady and transient heat conduction in a plane wall**

FIGURE 2–5

## Multidimensional Heat Transfer




**Two-dimensional heat transfer in a long rectangular bar**




FIGURE 2–6

**Heat transfer through the window of a hourse can be taken to be one-dimensional**

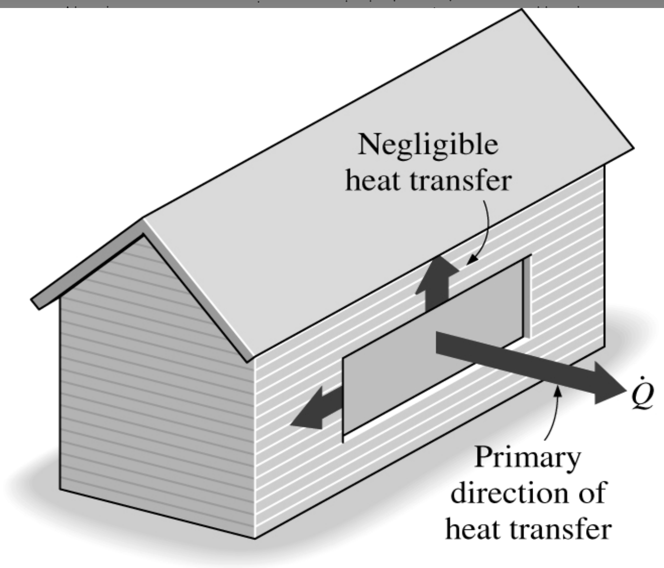
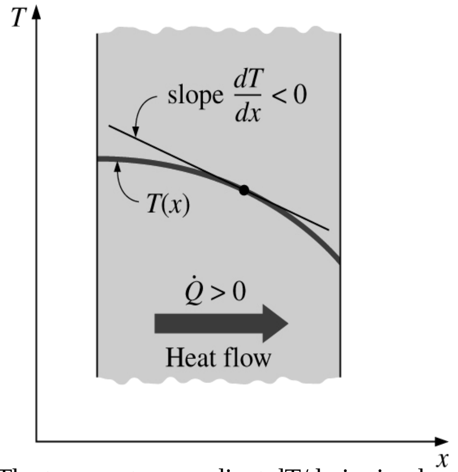


**FIGURE 2–7**



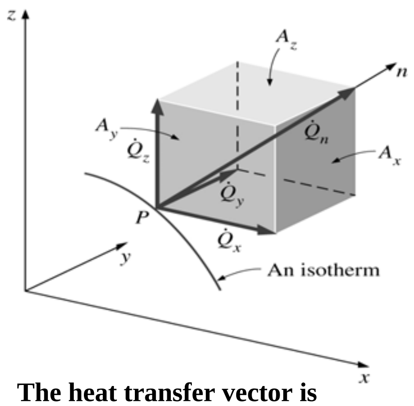

The temperature gradient dT/dx is simply the slope of the temperature curve on a T-x diagram

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} \qquad (W)$$





**FIGURE 2–8**




**The heat transfer vector is always normal to an isothermal surface and can be resolved into its components like any other vector**

**Fourier'law as**

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad (W)$$

**Expressed in term of its component as**

Q n = Q xi + Q yj + Q zk

**Heat transfer rate in the x,y,z direction**

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x} \qquad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y}$$

$$\dot{Q}_z = -kA_z\frac{\partial T}{\partial z}$$

FIGURE 2–9

**Heat Generation**

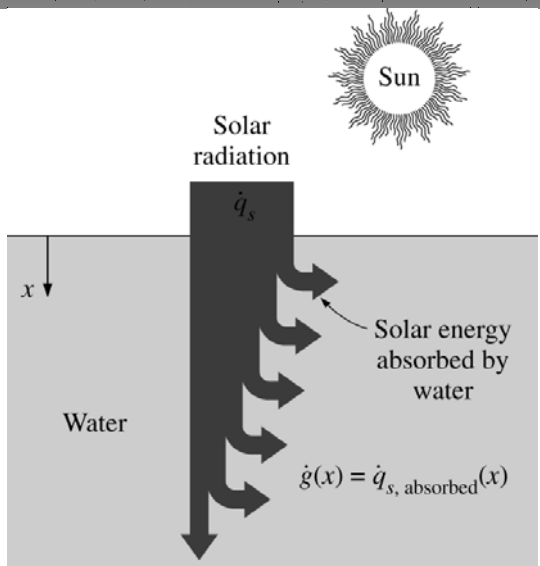
**Heat is generated in the heating coils of an electric range as a result of the conversion of electrical energy to heat**

9



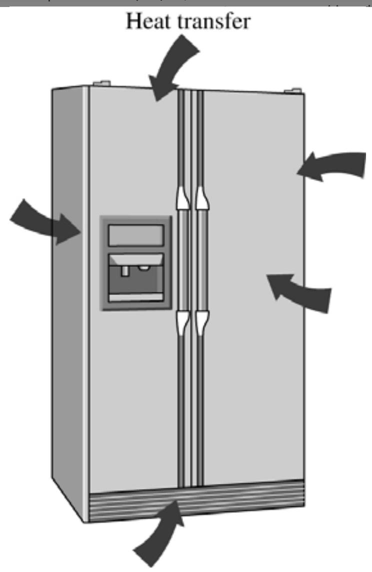
FIGURE 2–10

**The absorption of solar radiation by water can be treated as heat generation**




10

**FIGURE 2–11**

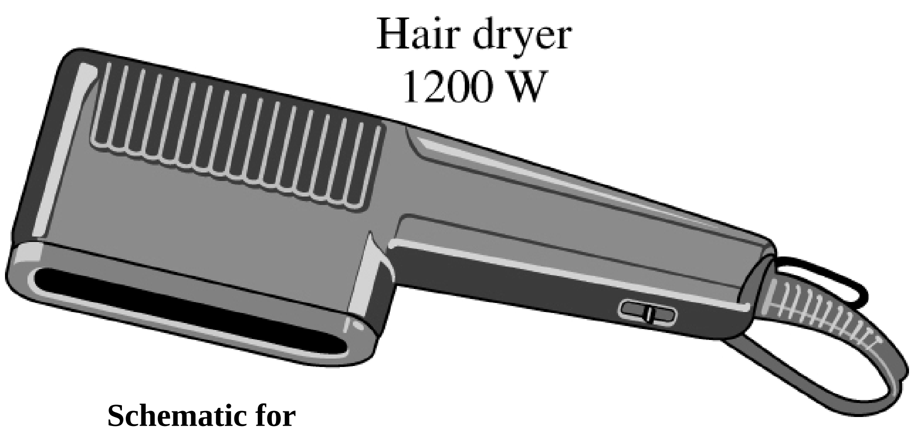



**Schematic for Example 2-1**



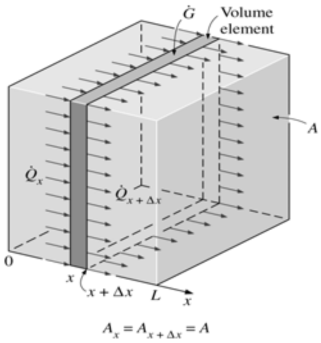


**FIGURE 2–12**




**Schematic for Example 2-2**

**FIGURE 2–13**




**One-dimensional heat conduction through a volume element in a large plane wall**

## 2.2 ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{element} = \frac{\Delta E_{element}}{\Delta t}$$

$$\Delta E_{element} = \rho C A \Delta x \left(T_{t+\Delta t} - T_t\right)$$
$$\dot{G}_{element} = \dot{g} A \Delta x$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{g} A \Delta x = \rho C A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$




**Continue FIGURE 2–13**

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t} \quad \Rightarrow \quad \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

*Constant conductivity* $\qquad \frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady-state* $(\partial/\partial t = 0)$ $\qquad \frac{d^2 T}{dx^2} + \frac{\dot{g}}{k} = 0$

*Transient, no heat generation* $(\dot{g} = 0)$ $\qquad \frac{d^2 T}{dx^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady-state, no heat generation* $(\partial/\partial t = 0$ *and* $\dot{g} = 0)$ $\qquad \frac{d^2 T}{dx^2} = 0$

**FIGURE 2–14**

General, one dimensional:

No generation — Steady-state

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k}^{\,0} = \frac{1}{\alpha}\frac{\partial T}{\partial t}^{\,0}$$

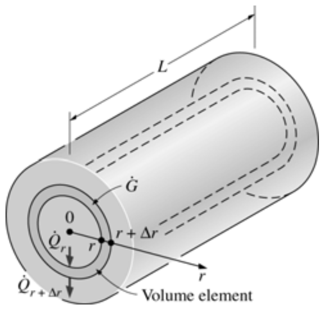
Steady, one-dimensional:

$$\frac{d^2 T}{dx^2} = 0$$





**FIGURE 2–15**




**One-dimensional heat conduction through a volume element in a long cylinder**

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{G}_{element} = \frac{\Delta E_{element}}{\Delta t}$$

$$\Delta E_{element} = \rho C A \Delta r (T_{t+\Delta t} - T_t)$$
$$\dot{G}_{element} = \dot{g} A \Delta r$$

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{g} A \Delta r = \rho C A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

## Continue FIGURE 2–15

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t} \quad \Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t}$$

*Constant conductivity* $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state* $(\partial/\partial t=0)$ $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$

*Transient no heat generation* $(\dot{g}=0)$ $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state no heat generation* $(\partial/\partial t=0 \text{ and } \dot{g}=0)$ $\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$





## FIGURE 2–16

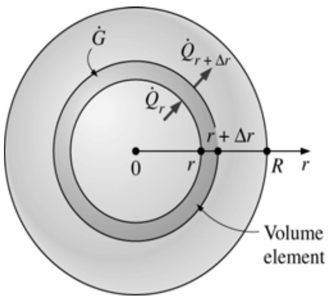
(*a*) The form that is ready to integrate

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

(*b*) The equivalent alternative form

$$r\frac{d^2T}{dr^2}+\frac{dT}{dr}=0$$

## FIGURE 2–17




**One-dimensional heat conduction through a volume element in a sphere**

*Variable conductivity* $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Constant conductivity* $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state* $(\partial/\partial t=0)$ $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$

*Transient no heat generation* $(\dot{g}=0)$ $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state no heat generation* $(\partial/\partial t=0, \dot{g}=0)$ $\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$




## Continue FIGURE 2–17

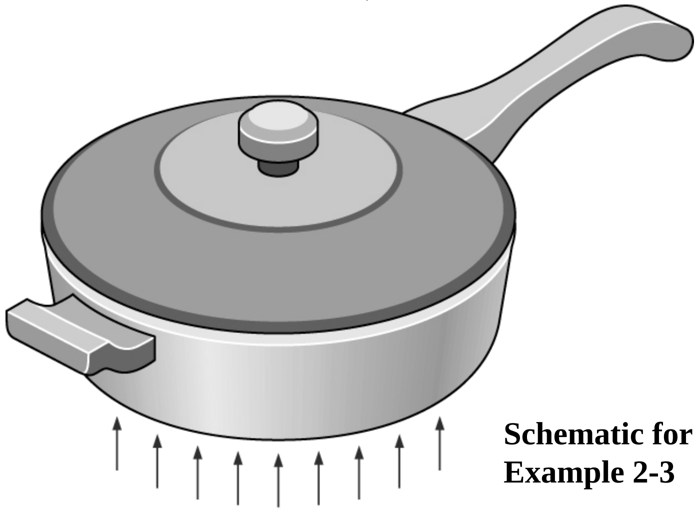
*Constant conductivity* $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{g}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state* $(\partial/\partial t=0)$ $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$

*Transient no heat generation* $(\dot{g}=0)$ $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

*Steady state no heat generation* $(\partial/\partial t=0, \dot{g}=0)$ $\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$

**FIGURE 2–18**




**Schematic for Example 2-3**




**FIGURE 2–19**

**Schematic for Example 2-4**

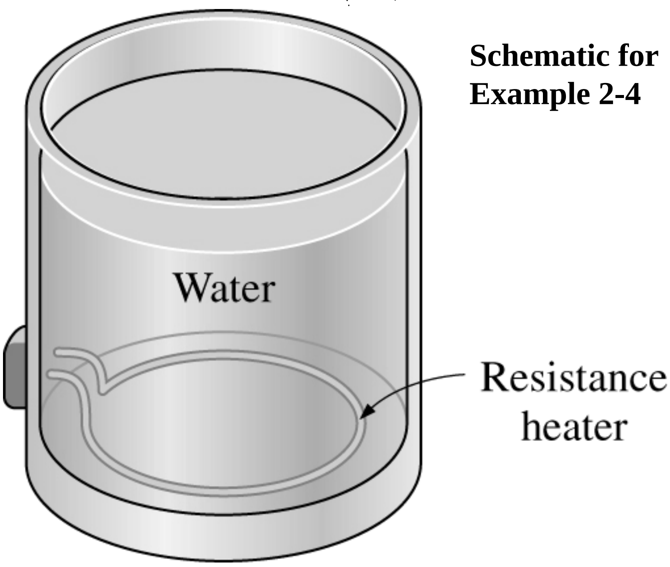
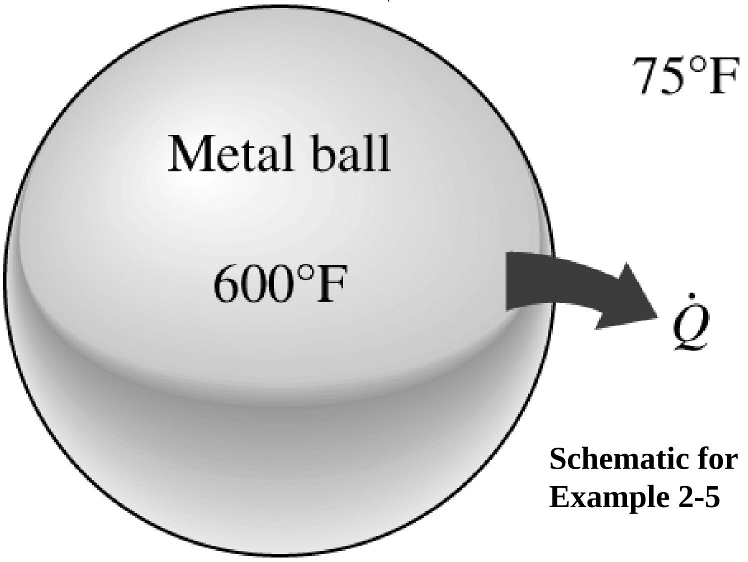


**FIGURE 2–20**




**Schematic for Example 2-5**

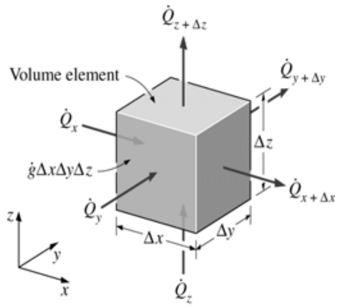
23



**FIGURE 2–21**

## 2.3 GENERAL HEAT CONDUCTION EQUATION

**Rectangular Coordinate**




**Three-dimensional heat conduction through a rectangular volume element**

Energy balance on this element

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{element} = \frac{\Delta E_{element}}{\Delta t}$$

$$\Delta E_{element} = \rho C A \Delta x \Delta y \Delta z \left(T_{t+\Delta t} - T_t\right)$$

$$\dot{G}_{element} = \dot{g} A \Delta x \Delta y \Delta z$$

24

## Continue FIGURE 2–21

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g}\Delta x \Delta y \Delta z = \rho C \Delta x \Delta y \Delta z \frac{(T_{t+\Delta t} - T_t)}{\Delta t}$$

$$-\frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x \Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C \frac{(T_{t+\Delta t} - T_t)}{\Delta t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

**In case constant thermal conductivity,it reduce to**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$




## FIGURE 2–22 Cont FIGURE 2–21

**Steady-state**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0$$

**Transient,no heat generate**

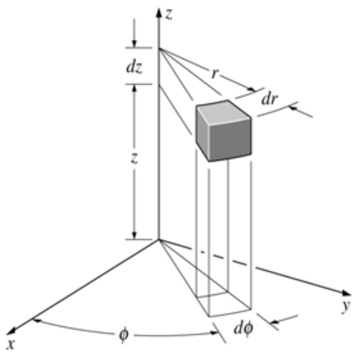
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

**Steady-stste,no heat generate**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

FIGURE 2–23

## Cylindrical Coordinates




A differential volume element in cylindrical coordinate.

**From**

$$x = r\cos\phi, \quad y = r\sin\phi, and \quad z = z$$

**After lengthy manipulations, we obtain**

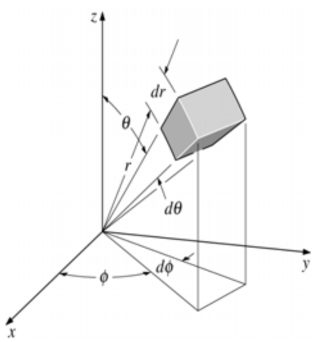
$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\phi}\left(kr\frac{\partial T}{\partial\phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{g}=\rho c\frac{\partial T}{\partial t}$$

27



FIGURE 2–24

## Spherical Coordinates




A differential volume element in spherical coordinate.

**From**

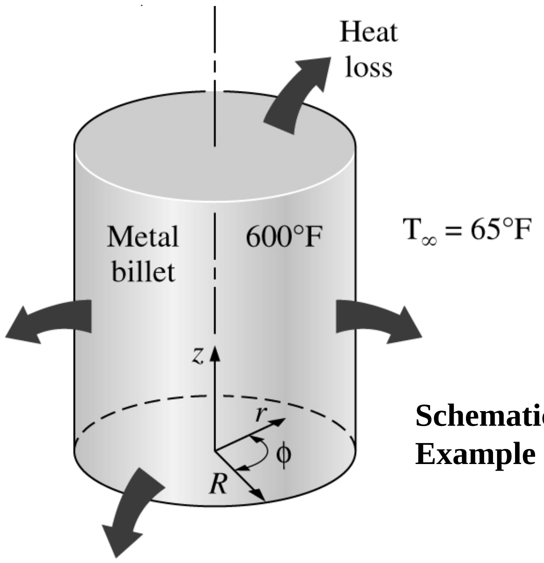
$$x = r\cos\phi\sin\theta, \quad y = r\sin\phi\sin\theta, \cos\theta$$

**After lengthy manipulations, we obtain**

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\dot{g}=\rho c\frac{\partial T}{\partial t}$$

28

**FIGURE 2–25**




**Schematic for Example 2-6.**

29



**FIGURE 2–26**

## 2.4 BOUNDARY AND INITIAL CONDITION

*The differential equation:*

$$\frac{d^2T}{dx^2} = 0$$

*General solution:*

$$T(x) = C_1x + C_2$$

Arbitrary constants

*Some specific solutions:*

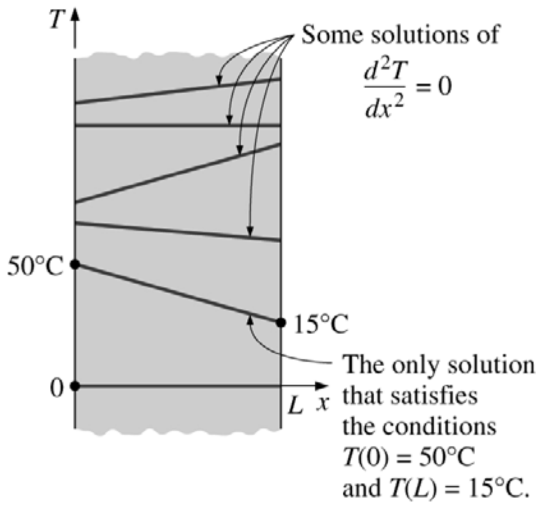
$$T(x) = 2x + 5$$
$$T(x) = -x + 12$$
$$T(x) = -3$$
$$T(x) = 6.2x$$
$$\vdots$$

**The general solution of a typical differential equationinvolves arbitary constants,andthus an infinite number of solutions.**

30

**FIGURE 2–27**

To describe a heat transfer problem completely,two boundary conditions must be given for each direction along which heat transfer is significant

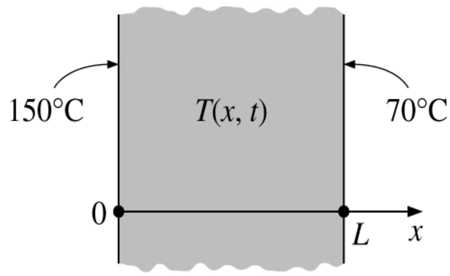



31



**FIGURE 2–28**

**Specified Temperature Boundary Condition**




$T(0, t) = 150°C$

$T(L, t) = 70°C$

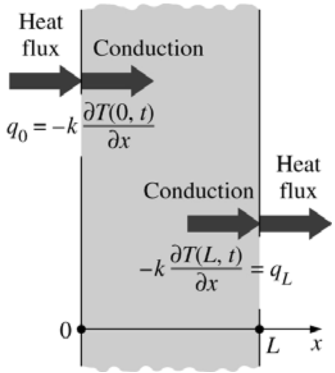
**$T(0,t) = T_1$**

**$T(L,t) = T_2$**

32

**FIGURE 2–29**

## Specified Heat Flux Boundary Condition



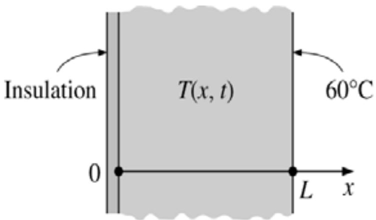

$$\dot{q} = -k \frac{\partial T}{\partial x} =$$ Heat flux in the positive x-direction (W/m²)





**FIGURE 2–30**

## Insulate Boundary




$$\frac{\partial T(0,t)}{\partial x} = 0$$

$$T(L,t) = 60°\text{C}$$

A plane wall with insulation and specified temperature boundary conditions

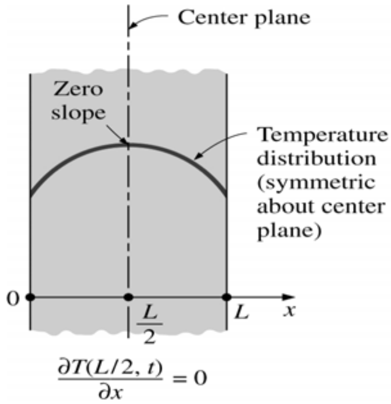
$$k \frac{\partial T(0,t)}{\partial x} = 0$$

*or*

$$\frac{\partial T(0,t)}{\partial x} = 0$$

FIGURE 2–31

**Thermal Symmetry**



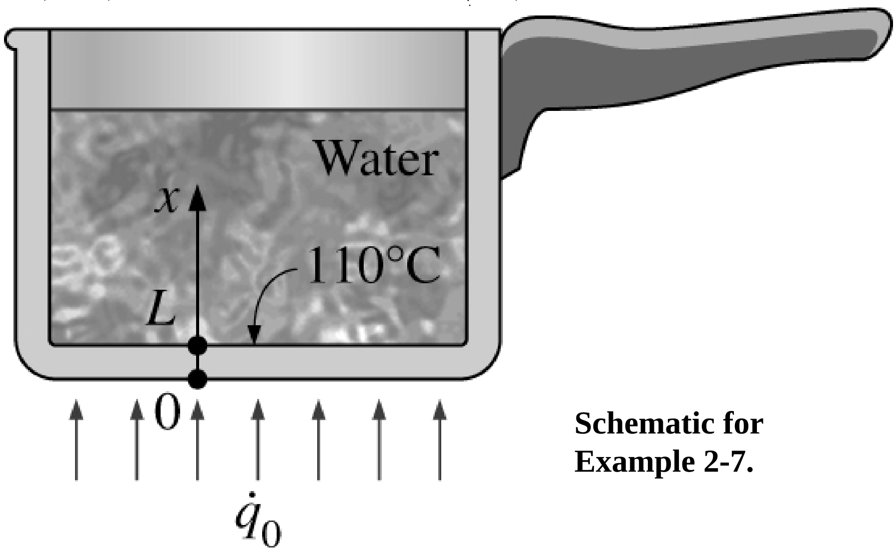

$$\frac{\partial T(0,t)}{\partial x} = 0$$

Thermal symmetry boundary condition at the center plane of a plane wall





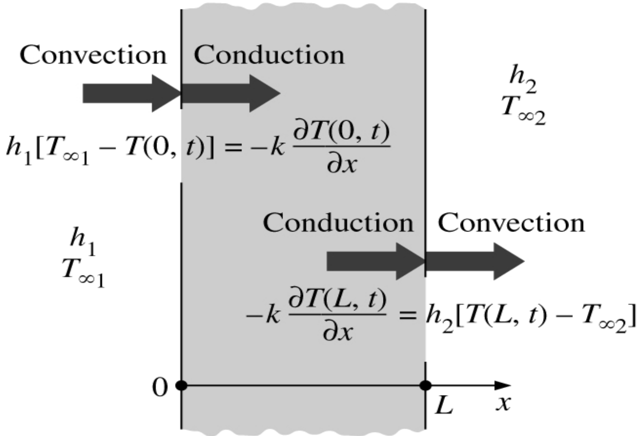
FIGURE 2–32




**Schematic for Example 2-7.**

FIGURE 2–33

**Convection Boundary Condition**

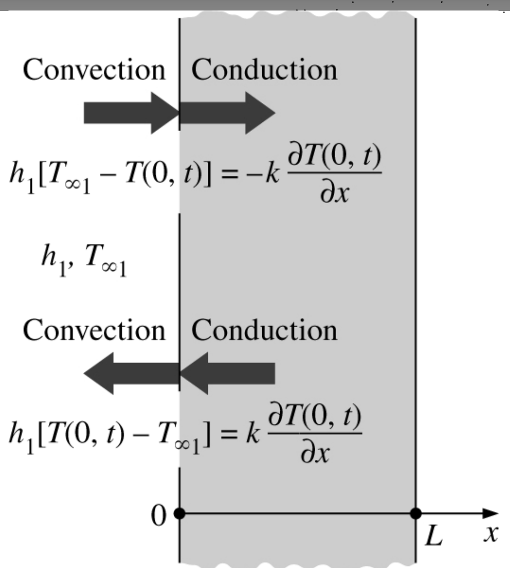



**Convection boundary condition on the two surface of a plane wall.**

37



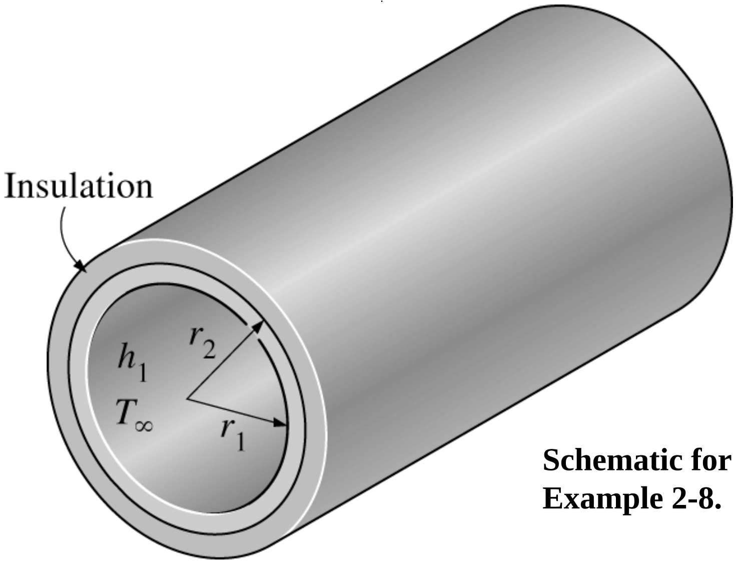
FIGURE 2–34




**The assumed direction of heat transfer at a boundary heat no effect on the boundary condition expression.**

38

**FIGURE 2–35**



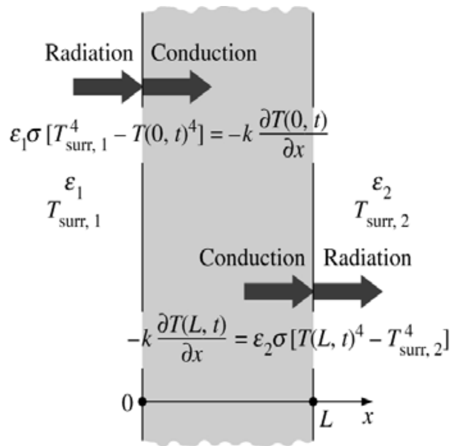

**Schematic for Example 2-8.**

39



**FIGURE 2–36**

**Radiation Boundary Condition**

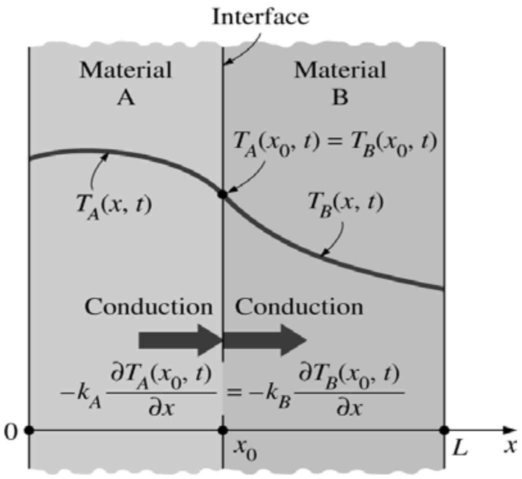



**Radiation boundary condition on both surfaces of a plane wall.**

40

FIGURE 2–37

**Interface Boundary Condition**



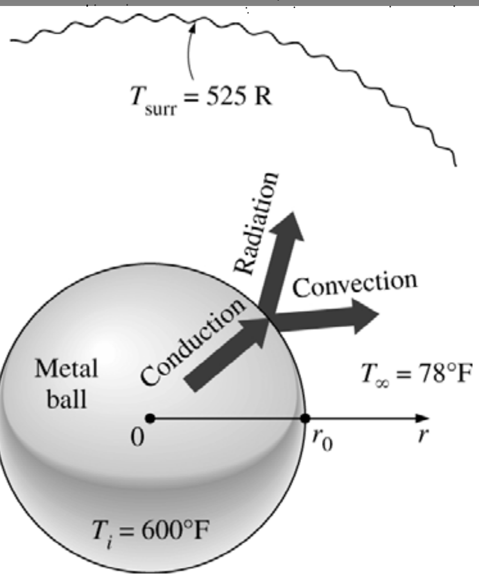

**Boundary condition at the interface of two bodies in perfect contact.**

41



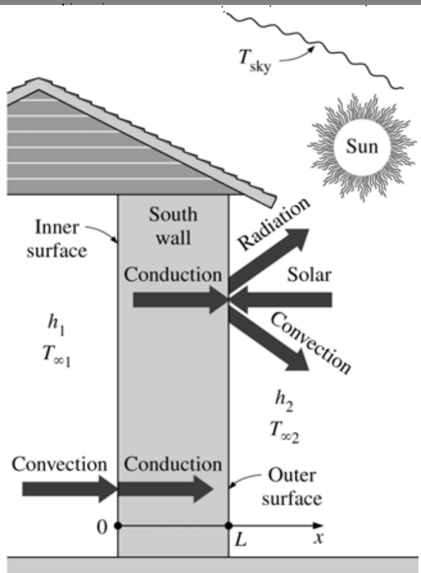
FIGURE 2–38

**Schematic for Example 2-9.**




42

**FIGURE 2–39**



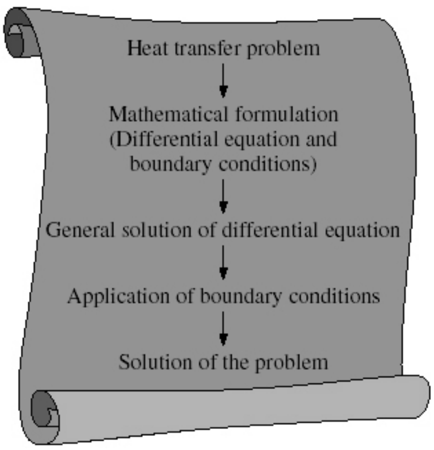

**Schematic for Example 2-10.**

43



**FIGURE 2–40**

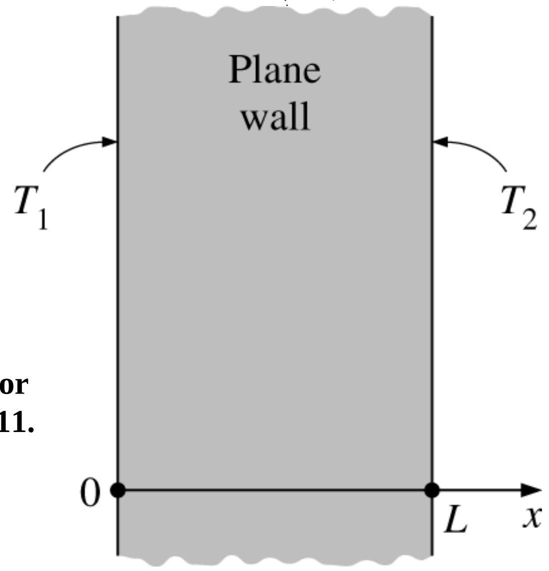
## 2.5 SOLUTION OF STEADY ONE-DIMENSIONAL HEAT CONDUCTION PROBLEMS




**Basic steps involved in the solution of heat transfer problems.**

44

**FIGURE 2–41**

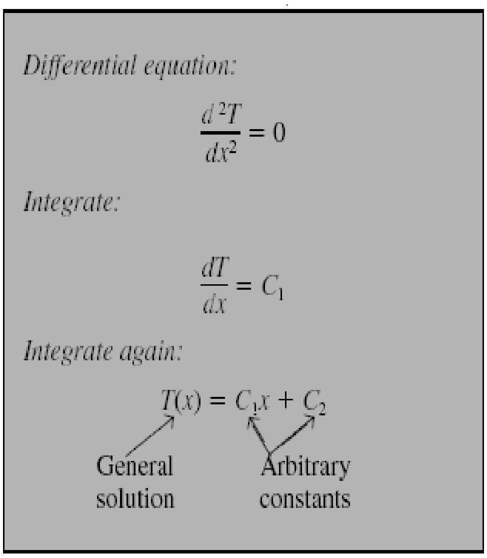



**Schematic for Example 2-11.**

45



**FIGURE 2–42**




**Obtaining the general solution of a simple second order differential equation by integration.**

46

## FIGURE 2–43

*Boundary condition:*

$$T(0) = T_1$$

*General solution:*

$$T(x) = C_1 x + C_2$$

*Applying the boundary condition:*

$$\underbrace{T(\underset{0}{x})}_{T_1} = C_1 \underset{0}{x} + C_2$$

*Substituting:*

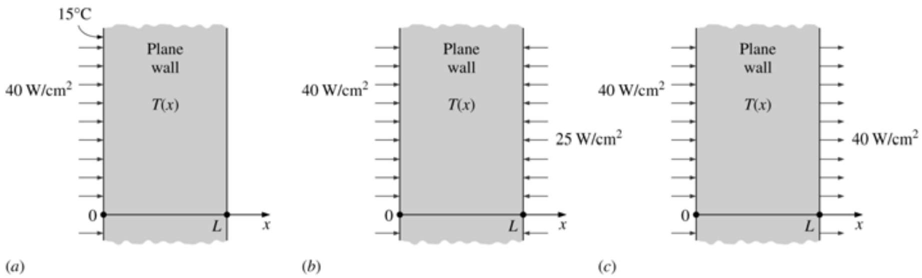
$$T_1 = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

It cannot involve $x$ or $T(x)$ after the boundary condition is applied.

**When apply a boundary condition to the general solution at a specified point, all occurrences of the dependent and independent variables should be replaced by their specified values at that point.**




## FIGURE 2–44




**Schematic for Example 2-12.**

## FIGURE 2–45

*Differential equation:*

$$T''(x) = 0$$

*General solution:*

$$T(x) = C_1 x + C_2$$

(*a*) *Unique solution:*

$$\left.\begin{aligned} -kT'(0) &= \dot{q}_0 \\ T(0) &= T_0 \end{aligned}\right\} T(x) = -\frac{\dot{q}_0}{k}x + T_0$$

(*b*) *No solution:*

$$\left.\begin{aligned} -kT'(0) &= \dot{q}_0 \\ -kT'(L) &= \dot{q}_L \end{aligned}\right\} T(x) = \text{None}$$

(*c*) *Multiple solutions:*

$$\left.\begin{aligned} -kT'(0) &= \dot{q}_0 \\ -kT'(L) &= \dot{q}_0 \end{aligned}\right\} T(x) = -\frac{\dot{q}_0}{k}x + \underset{\uparrow}{C_2}$$

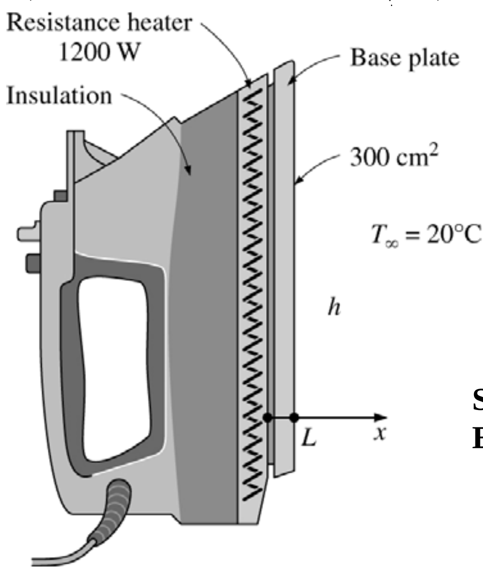
Arbitrary

**A boundary-value problem may have a uniquesolution,infinitely many solutions, or no solutions at all.**



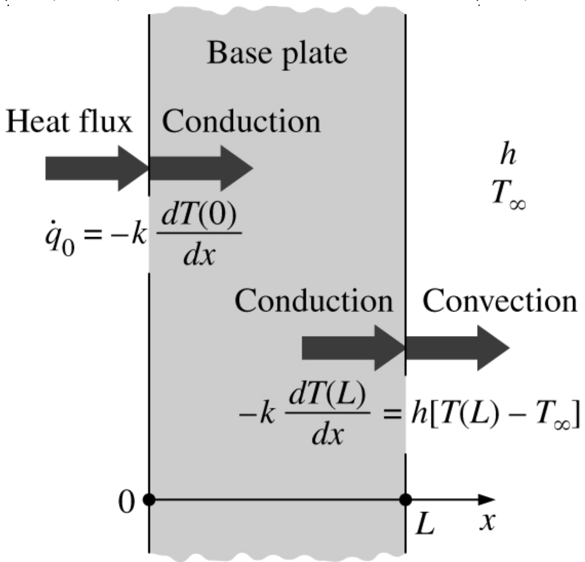


## FIGURE 2–46




**Schematic for Example 2-13.**

FIGURE 2–47

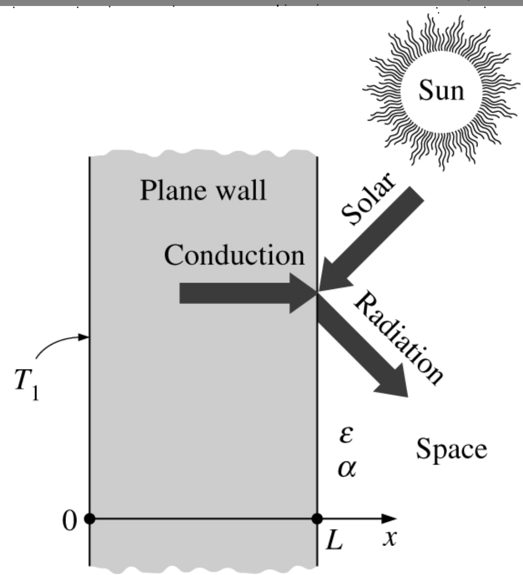



**The boundary conditions on the base plate of the iron discussed in example 2-13.**

51



FIGURE 2–48




**Schematic for Example 2-14.**

52

## FIGURE 2–49

(1) *Rearrange the equation to be solved:*

$$T_L = 310.4 - 0.240975\left(\frac{T_L}{100}\right)^4$$

The equation is in the proper form since the left side consists of $T_L$ only.

(2) *Guess the value of $T_L$, say 300 K, and substitute into the right side of the equation. It gives*

$$T_L = 290.2 \text{ K}$$

(3) *Now substitute this value of $T_L$ into the right side of the equation and get*

$$T_L = 293.1 \text{ K}$$

(4) *Repeat step (3) until convergence to desired accuracy is achieved. The subsequent iterations give*

$$T_L = 292.6 \text{ K}$$
$$T_L = 292.7 \text{ K}$$
$$T_L = 292.7 \text{ K}$$

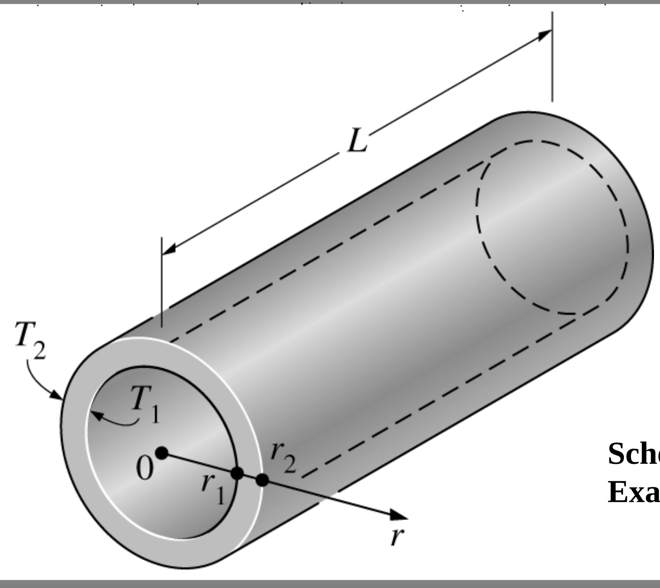
Therefore, the solution is $T_L = 292.7$ K. The result is independent of the initial guess.

**A sample method of solving a nonlinear equation is to arrange the equation such that the unknown is alone on the left side while everythingelse is on the right side, and to interate after an initial guess until convergence.**

53



## FIGURE 2–50




**Schematic for Example 2-15.**

54

**FIGURE 2–51**

*Differential equation:*

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

*Integrate:*

$$r\frac{dT}{dr} = C_1$$

Divide by $r$ ($r \neq 0$):

$$\frac{dT}{dr} = \frac{C_1}{r}$$

*Integrate again:*

$$T(r) = C_1 \ln r + C_2$$

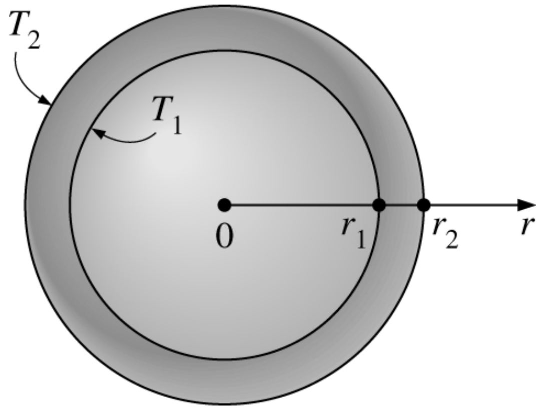
*which is the general solution.*

**Basic steps involved in the solution of the steady one-dimensional heat conduction in cylindrical coordinates.**



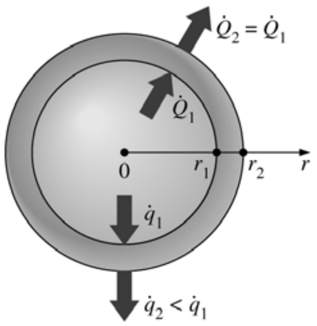


**FIGURE 2–52**




**Schematic for Example 2-16**

**FIGURE 2–53**




$$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.14 \text{ kW}}{4\pi(0.08 \text{ m})^2} = 337.5 \text{ kW/m}^2$$

$$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.14 \text{ kW}}{4\pi(0.10 \text{ m})^2} = 216.0 \text{ kW/m}^2$$

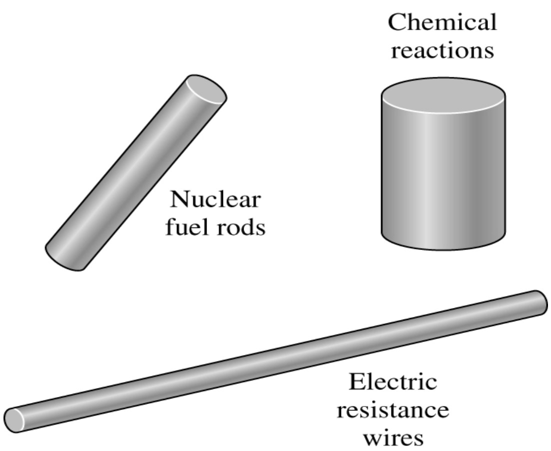
**During steady one-dimensional heat conduction in a spherical (or cylindrical) container,the total rate of heat transfer remains constant, but the heat flux decreases with increasing radius.**





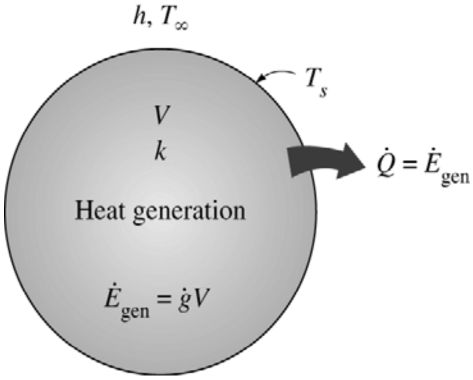
**FIGURE 2–54**

## 2.6 HEAT GENERATION IN A SOLID




**Heat generation in solid is commonly encountered in practice.**

## FIGURE 2–55




At steady conditions, the entire heat generated in a solid must leave the solid through its outer surface.

**from**

$$\dot{Q} = hA_s(T_s - T_\infty) = \dot{g}V$$

**We obtain**

$$T_s = T_\infty + \frac{\dot{g}V}{hA_s}$$

**Reduces to**

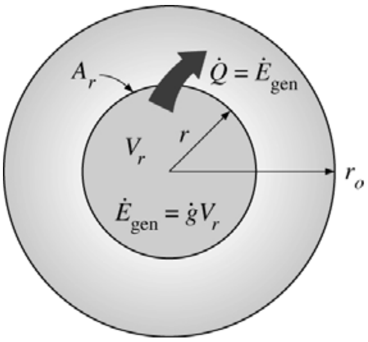
$$T_{s,\,planewall} = T_\infty + \frac{\dot{g}L}{h}$$

$$T_{s,cylinder} = T_\infty + \frac{\dot{g}r_0}{2h}$$

$$T_{s,sphere} = T_\infty + \frac{\dot{g}r_0}{3h}$$




## FIGURE 2–56




**Heat conducted through a cylindrical shell of radius r is equal to the heat generated within a shell.**

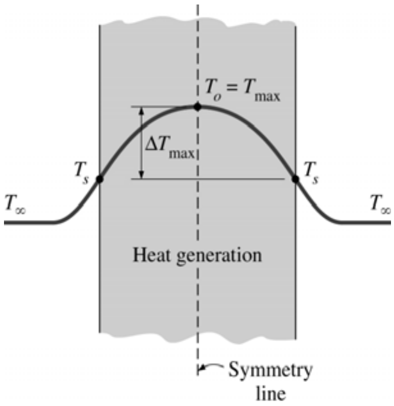
**from**

$$-kA_r\frac{dT}{dr} = \dot{g}V_r \longrightarrow dT = -\frac{\dot{g}}{2k}rdr$$

**Int. from r=0 to r=$r_0$ where T(0)=$T_0$ and T($r_0$)=$T_s$**

$$\Delta T_{\max,cylinder} = T_0 - T_s = \frac{\dot{g}r_0^2}{4k}$$

**FIGURE 2–57**



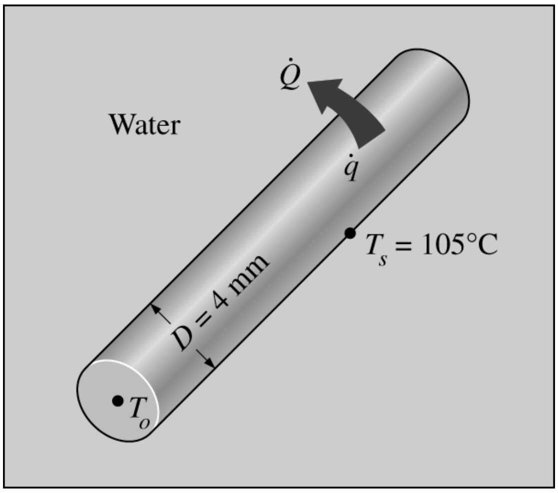

$$T_{center} = T_0 = T_s + \Delta T_{max}$$

**The maximum Temperature in a symmetrical solid with uniform heat generation occurs at its center.**



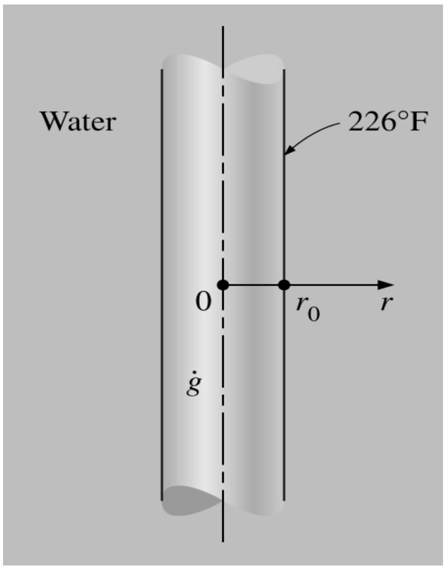


**FIGURE 2–58**




**Schematic for Example 2-17.**

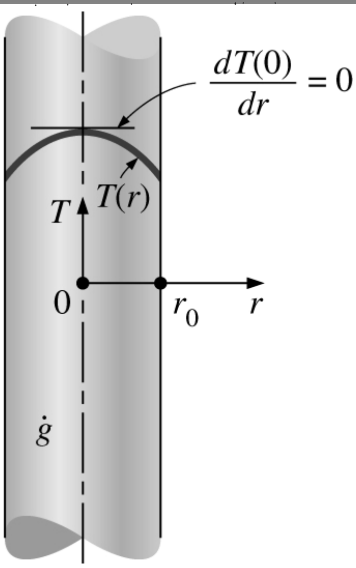
**FIGURE 2–59**




**Schematic for Example 2-18.**

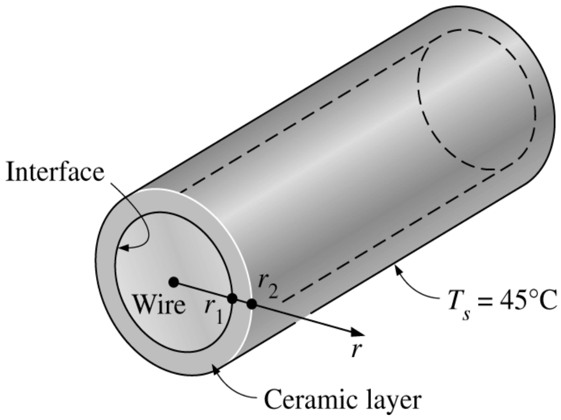



**FIGURE 2–60**




**The thermal stmmetry condition at the centerline of a wire in which heat is generated uniformly.**

FIGURE 2–61



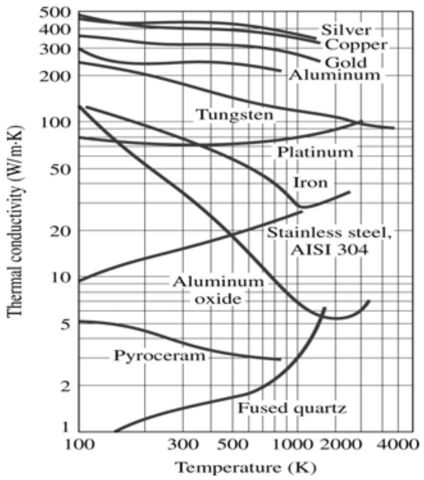

**Schematic for Example 2-19.**





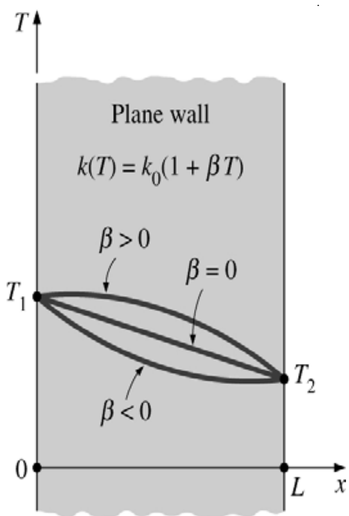
FIGURE 2–62

## 2.7 VARIABLE THERMAL CONDUCTIVITY,k(T)




**Variation of the thermal conductivity of some solids with temperature.**

## FIGURE 2–63



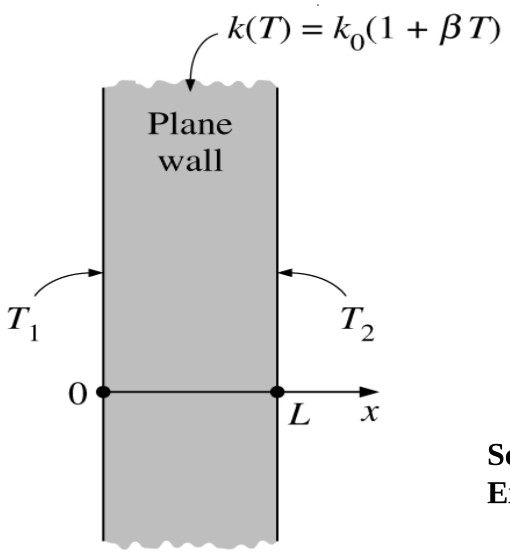

β Is called the **temperature coefficient of thermal conductivity.**

**The variation of temperature in a plane wall during steady one-dimensional heat conduction for the case of constant and variable thermal conductivity.**

67



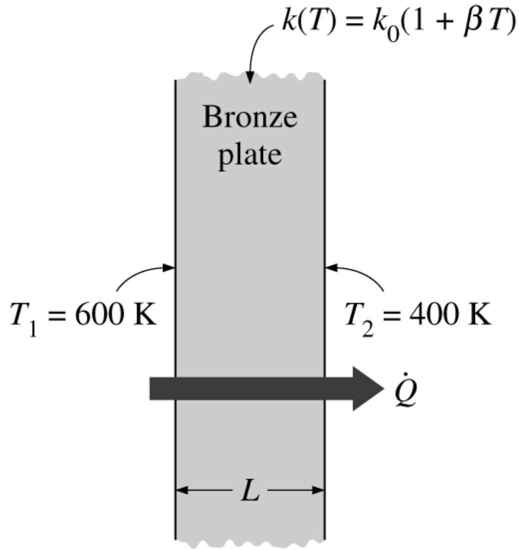
## FIGURE 2–64




**Schematic for Example 2-20.**

68

**FIGURE 2–65**

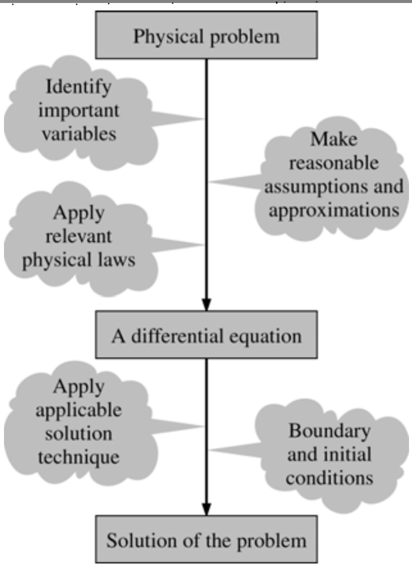



**Schematic for Example 2-21.**

69



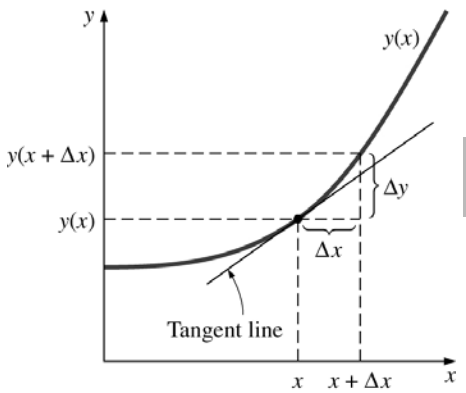
**FIGURE 2–66**




**Mathematical modeling of physical problems.**

70

**FIGURE 2–67**

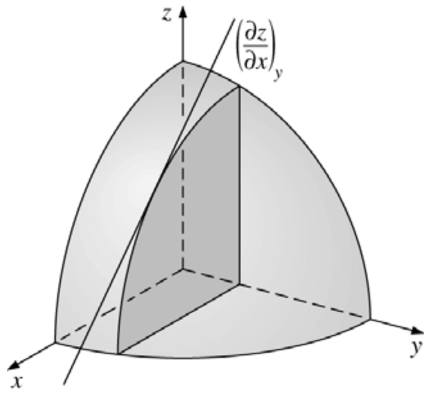



$$y'(x) = \frac{dy(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x}$$

**The derivative of a function at a point represents the slope of the tangent line of the function at the point.**




**FIGURE 2–68**




$$\frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{y(x+\Delta x, t) - y(x,t)}{\Delta x}$$

$$\frac{\partial y}{\partial t} = \lim_{\Delta t \to 0} \frac{y(x, t+\Delta t) - y(x,t)}{\Delta t}$$

**Graphical representation of partial derivative** $\partial z / \partial x$

**FIGURE 2–69**

$$\int dy = y + C$$

$$\int y'\, dx = y + C$$

$$\int y''\, dx = y' + C$$

$$\int y'''\, dx = y'' + C$$

$$\int y^{(n)}\, dx = y^{(n-1)} + C$$

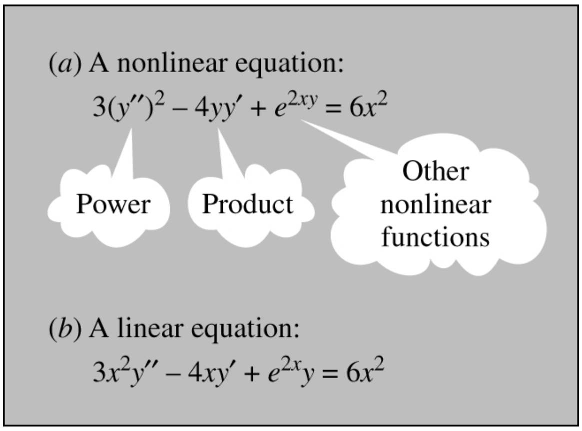
**Some indefinite integrals that involve derivatives.**

73



**FIGURE 2–70**

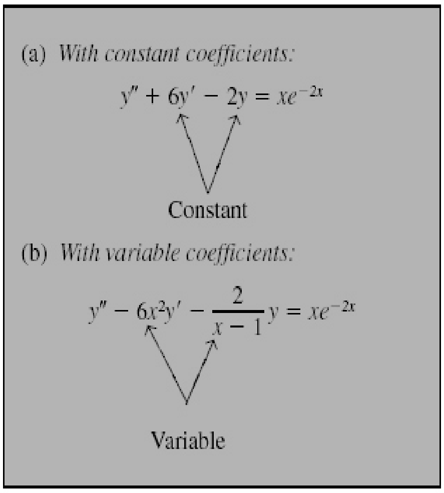
**Classification of Differential Equations**




**Adifferential equation that is (a)nonlinear and (b)linear.When checking for linearity,we examine the dependent variable only.**

74

## FIGURE 2–71




**A general form of differential equation**

$$y^{(n)} + f_1(x)y^{(n-1)} + \ldots + f_{n-1}(x)y' + f_n(x)y = R(x)$$

**A differential equation with (a) constant coefficients and (b) variable coefficients.**




## FIGURE 2–72

### Solution of Differential Equations

(a) *An algebraic equation:*

$$y^2 - 7y - 10 = 0$$

*Solution:* $y = 2$ and $y = 5$

(b) *A differential equation:*

$$y' - 7y = 0$$

*Solution:* $y = e^{7x}$

**Unlike those of algebraic equations.the solution of differential equations are typically functions instead of discrete values.**

**FIGURE 2–73**

Function: $f = 3e^{-2x}$

*Differential equation:* $y'' - 4y = 0$

*Derivatives of f:*

$$f' = -6e^{-2x}$$
$$f'' = 12e^{-2x}$$

Substituting into $y'' - 4y = 0$:

$$f'' - 4f \overset{?}{=} 0$$
$$12e^{-2x} - 4 \times 3e^{-2x} \overset{?}{=} 0$$
$$0 = 0$$

Therefore, the function $3e^{-2x}$ is a solution of the differential equation $y'' - 4y = 0$.

**Verifying that a given function is a solution of a differential equation.**

77