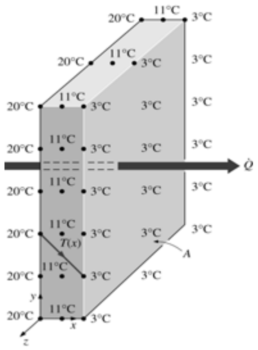
FIGURE 3–1

STEADY HEAT CONDUCTION IN PLANE WALLS



The energy balance for the wall can be expressed as

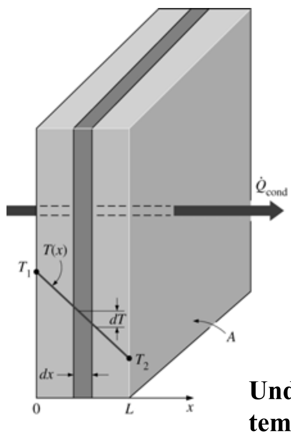
$$\begin{pmatrix}\text{Rate of}\\ \text{heat trans fer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat trans fer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

or

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

Heat flow through a wall is one dimensional when the temperature of the wall varies in one direction only

$x=0$
FIGURE 3–2



Separating the variables in the above equation and integrating from where $x=0$ $T(0)=T_1$ **, to** $x=L$ **, where** $T(L)=T_2$ **we get**

$$\int_{x=0}^{L} \dot{Q}_{cond,wall}\, dx = -\int_{T=T_1}^{T_2} kAdT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \quad (W)$$

Under steady conditions, the temperature distribution in a plane wall is a straight line.

FIGURE 3–2

The Thermal Resistance Concept

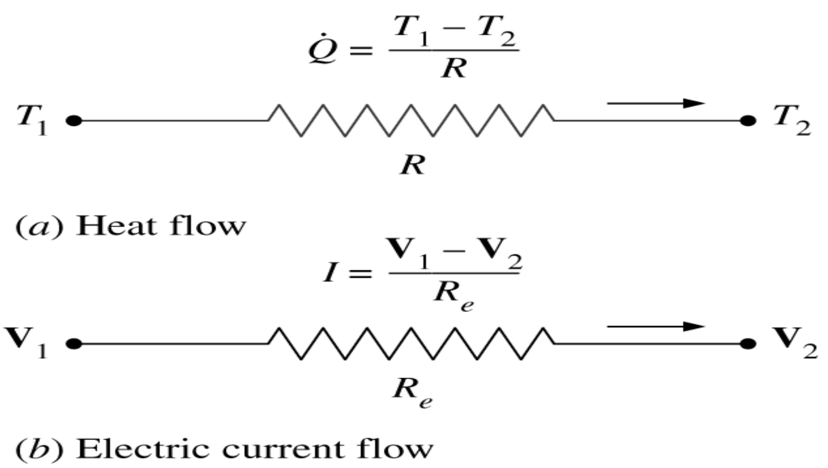
$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \quad (W)$$

Where

$$R_{wall} = \frac{L}{kA} \qquad (°C/W) \qquad \textbf{thermal resistance}$$

3

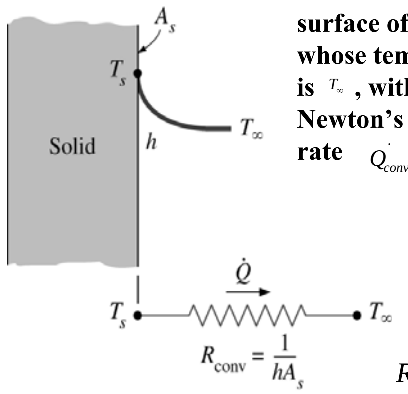
FIGURE 3–3



Analogy between thermal and electrical resistance concepts.

4

FIGURE 3–4



Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate $\dot{Q}_{conv} = hA_s(T_s - T_\infty)$ can be rearranged as

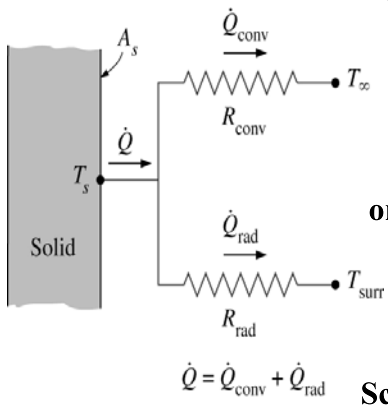
$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}}$$

Where

$$R_{wall} = \frac{1}{hA_s} \quad (°C/W)$$ thermal resistance

Schematic for convection resistance at a surface.

FIGURE 3–5



$$\dot{Q}_{rad} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right) = h_{rad} A_s\left(T_s - T_{sur}\right)$$

$$= \frac{T_s - T_{sur}}{R_{rad}} \Rightarrow R_{rad} = \frac{1}{h_{rad} A_s}$$

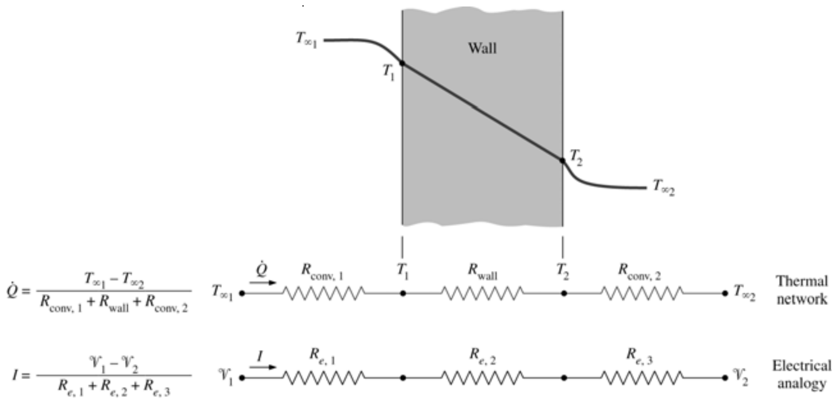
thermal resistance

or, the radiation resistance

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s\left(T_s - T_{surr}\right)} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$

Schematic for convection and radiation resistance at a surface.

FIGURE 3–6



The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electric analogy.

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FIGURE 3–6

The Resistance Network

Under steady conditions we have

$$\begin{pmatrix}\text{Rate of}\\ \text{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat convection}\\ \text{from the wall}\end{pmatrix}$$

or

$$\dot{Q}_{conv} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

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FIGURE 3–6

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{wall}} = \frac{T_2 - T_{\infty 2}}{R_{conv,2}}$$

Adding the numerators and denominators yields (Fig.3-7)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad (W)$$

FIGURE 3–7

If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = c$$

then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = c$$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

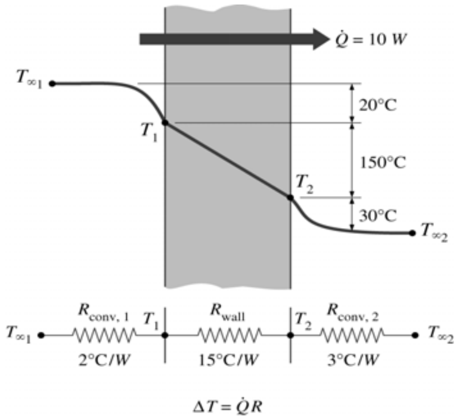
and

$$\frac{1 + 2 + 5}{4 + 8 + 20} = 0.25$$

<u>Thermal Resistance Network.</u>

A useful mathematical identity.

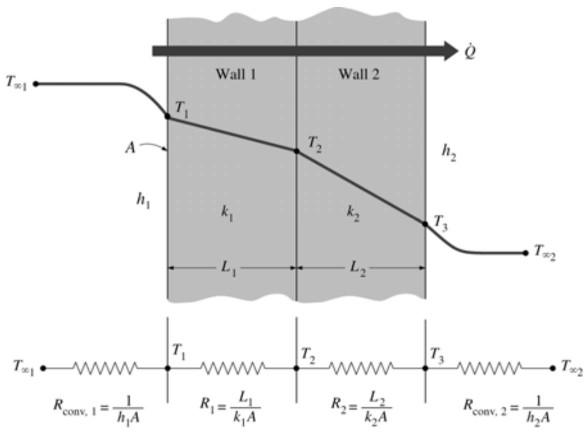
FIGURE 3–8



The temperature drop across a layer is proportional to its thermal resistance.

$$R_{total} = R_{conv1} + R_{wall} + R_{conv2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad {}^0C/W$$

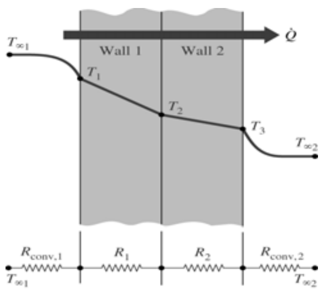
FIGURE 3–9



The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

FIGURE 3–10



To find T_1: $\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$

To find T_2: $\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1}$

To find T_3: $\dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{conv,2}}$

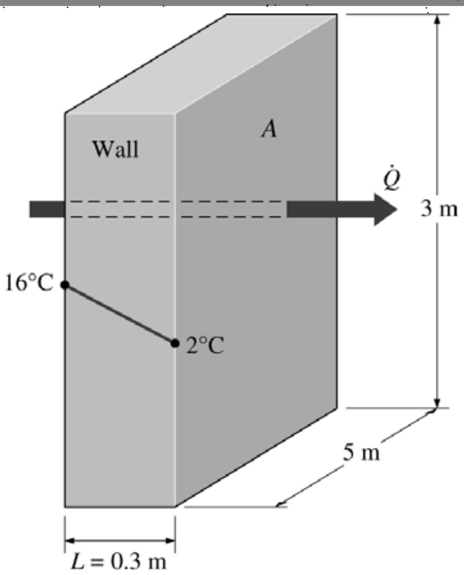
Once $\dot{Q}$ is known, an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{total,i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{wall,1}} = \frac{T_{\infty 1} - T_2}{(1/h_1 A) + (L_1/k_1 A)}$$

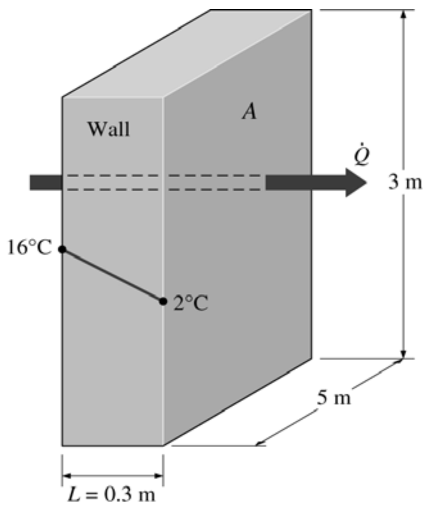
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

FIGURE 3–11



Schematic for Example 3-1.

FIGURE 3–11



Schematic for Example 3-1.

Assumptions

1. steady state heat transfer
2. (1-D) heat transfer
3. thermal conductivity is constant

Properties

1. the thermal conductivity is given to be k=0.9 W/m*C

Analysis

A = 3m x 5m = 15 m^2

FIGURE 3–11

Example 3-1.

SOLUTION

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\,W/m.°C)\left(15\,m^2\,\frac{(16-2)°C}{0.3m}\right) = 630W$$

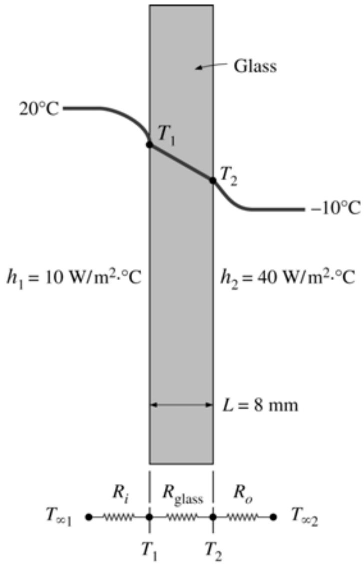
$$\dot{Q} = \frac{\Delta T_{wall}}{R_{wall}}$$

where $R_{wall} = \frac{L}{kA} \quad (°C/W)$

$$= \frac{0.3m}{(0.9\,W/m.°C)(15\,m^2)} = 0.02222°C/W$$

$$\dot{Q} = \frac{(16-2)°C}{0.02222\ °C/W} = 630\ W$$

FIGURE 3–12



Schematic for Example 3-2.

Assumptions

1.steady state heat transfer

2. (1-D) heat transfer

3. thermal conductivity is constant

Properties

1. the thermal conductivity is given to be k=0.78 W/m*C

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- **solution**

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10W/m^2\,{}^0C)(1.2m^2)} = 0.08333\ {}^0C/W$$

$$R_{glass} = \frac{L}{kA} = \frac{0.008m}{(0.78W/m^0C)(1.2m^2)} = 0.00855\ {}^0C/W$$

$$R_o = R_{conv2} = \frac{1}{h_2 A} = \frac{1}{(40W/m^2\,{}^0C)(1.2m^2)} = 0.02083\ {}^0C/W$$

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv2} = 0.08333 + 0.00855 + 0.02083 = 0.1127\ {}^0C/W$$

Then Steady rate of heat transfer through the window become

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[20 - (-10)]}{0.1127} = 266\ W$$

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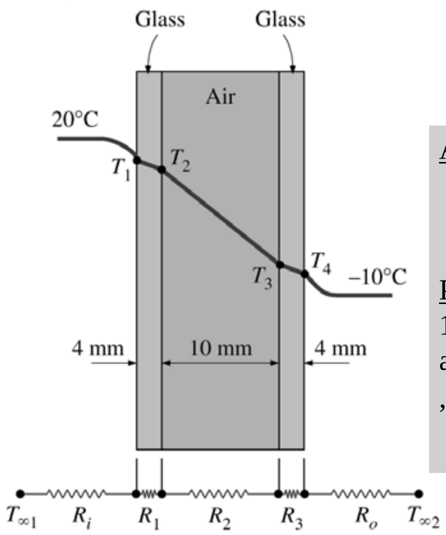
Knowing rate of heat transfer , the inner surface temperature of the winder can be determine from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$$

$$T_1 = T_\infty - \dot{Q}R_{conv,1} = 20 - (266)(0.08333) = -2.2^0 C$$

Discussion – Note that the inner surface temperature of the window glass will be even $-2.2°C$ though the temperature of the air in the room is maintained at $20°C$ Such low surface temperature are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

FIGURE 3–13



Schematic for Example 3-3.

Assumptions
1.steady state heat transfer
2. (1-D) heat transfer
3. thermal conductivity is constant

Properties
1. the thermal conductivity of glass and air space is given to be k=0.78 ,0.026 W/m*C

solution

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10W/m^{2\,0}C)(1.2m^2)} = 0.08333\ {}^0C/W$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.004m}{(0.78W/m^0C)(1.2m^2)} = 0.00427\ {}^0C/W$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.01m}{(0.026W/m^0C)(1.2m^2)} = 0.3205\ {}^0C/W$$

$$R_o = R_{conv2} = \frac{1}{h_2 A} = \frac{1}{(40W/m^{2\,0}C)(1.2m^2)} = 0.02083\ {}^0C/W$$

$$R_{total} = R_{conv,1} + R_{glass,1} + R_{air} + R_{glass,2} + R_{conv2}$$

$$= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 = 0.4332\ {}^0C/W$$

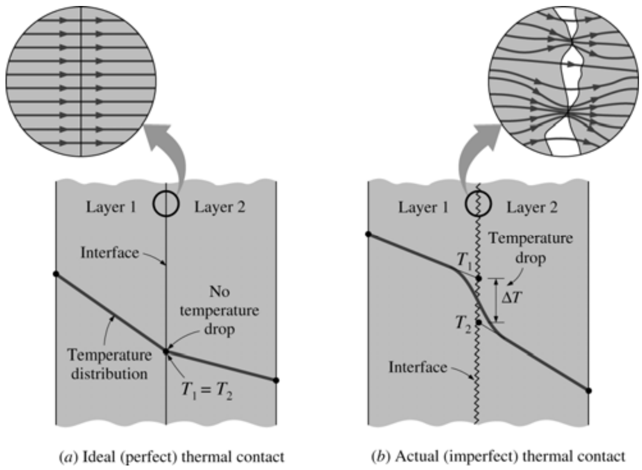
Then Steady rate of heat transfer through the window become

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20-(-10)]}{0.4332} = 69.2\ W$$

Knowing rate of heat transfer , the inner surface temperature of the winder can be determine from

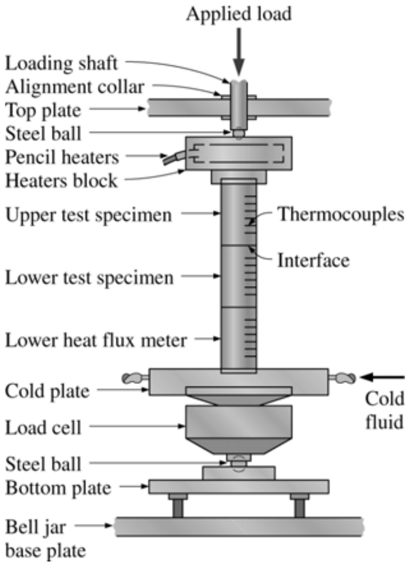
$$T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 20 - (69.2)(0.08333) = 14.2^0C$$

FIGURE 3–14



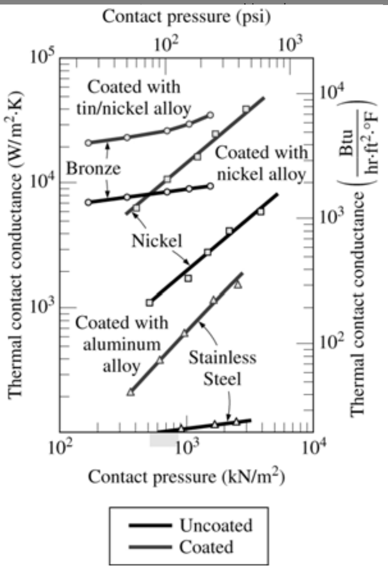
Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

FIGURE 3–15



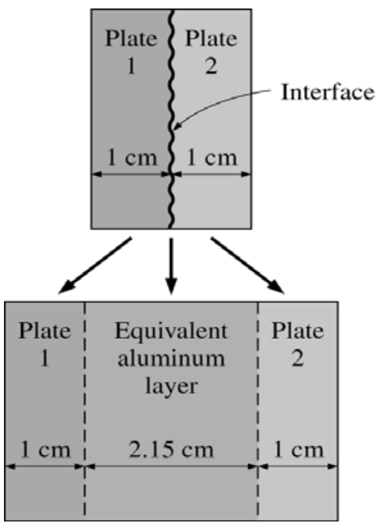
A typical experimental setup for the determination of thermal contact resistance.

FIGURE 3–16



Effect of metallic coatings on thermal contact conductance.

FIGURE 3–17



Schematic for Example 3-4.

<u>Assumptions</u>
1.steady state heat transfer
2. (1-D) heat transfer
3. thermal conductivity is constant

<u>Properties</u>
1. the thermal conductivity of aluminum at room temperature is k=237 W/m*C

FIGURE 3–11

Example 3-4. SOLUTION

The thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11000 W/m.°C} = 0.909x10^{-4} m^2.^0C/W$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity, Setting $R = R_{cr}$ **,the equivalent thickness is determined from the relation above to be**

$$L = kR_c = (237 W/m^0.C)(0.909 x10^{-4} m^2.^0C/W)$$

$$= 0.0215m = 2.15cm$$

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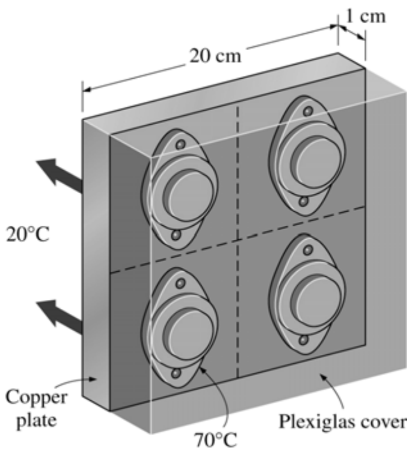
FIGURE 3–18



Schematic for Example 3-5.

Properties

-The thermal conductivity of copper is given to be k=386 W/m*C.

-The contact conductance

$h_c = 42{,}000 \quad W/m^2$

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FIGURE 3–18

Solution

$$R_{interface} = \frac{1}{h_c A_c} = \frac{1}{(42000W/m^{2\,0}C)(8x10^{-4}m^2)} = 0.03^0 C/W$$

$$R_{plate} = \frac{L}{kA} = \frac{0.01m}{(386W/m^0C)(0.01m^2)} = 0.0026\ ^0C/W$$

$$R_{conv} = \frac{1}{h_0 A} = \frac{1}{(25W/m^{2\,0}C)(0.01m^2)} = 4.0^0 C/W$$

$$R_{total} = R_{interface} + R_{plate} + R_{ambient}$$

$$= 0.03 + 0.0026 + 4.0^0 C/W$$

FIGURE 3–18

Solution

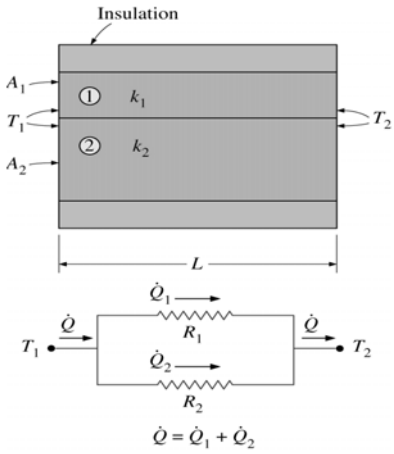
The rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{(70-20)°C}{4.0326°C/W} = 12.4W$$

The temperature jump at the interface is determined from

$$\Delta T_{interface} = \dot{Q}\,R_{interface} = (12.4W)(0.03°C/W) = 0.37°C$$

FIGURE 3–19



Thermal resistance network for two parallel layers.

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}} \Rightarrow R_{total} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

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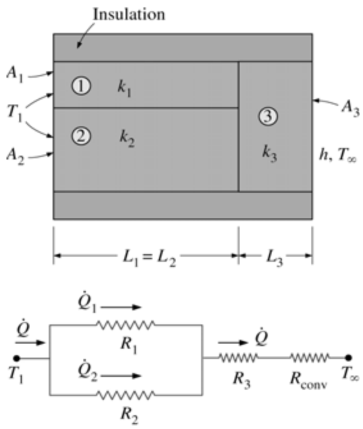
FIGURE 3–20



Thermal resistance network for combined series-parallel arrangement.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, R_2 = \frac{L_2}{k_2 A_2} R_3 = \frac{L_3}{k_3 A_3}, R_{conv} = \frac{1}{hA_3}$$

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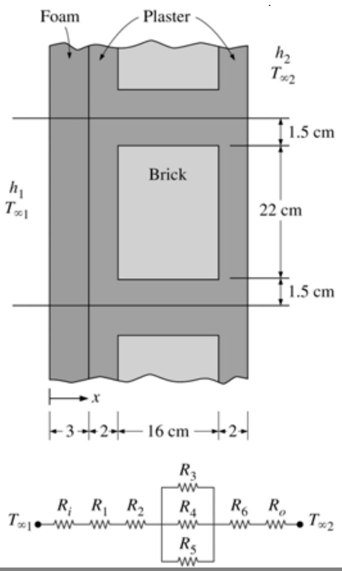
FIGURE 3–21



Schematic for Example 3-6.

Assumptions

1. steady state heat transfer
2. (1-D) heat transfer
3. thermal conductivity is constant

Properties

1. the thermal conductivity are given to be k=0.72 W/m*C for plaster layer, and k=0.026 W/m*C **for the rigid foam**

solution

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10W/m^{2\,0}C)(0.25x1m^2)} = 0.4^0C/W$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.03m}{(0.026W/m^0C)(0.25x1m^2)} = 4.6\,^0C/W$$

$$R_2 = R_6 = R_{plaster,side} = \frac{L}{kA} = \frac{0.02m}{(0.22W/m^0C)(0.25x1m^2)} = 0.36\,^0C/W$$

$$R_3 = R_5 = R_{plaster,center} = \frac{L}{kA} = \frac{0.16m}{(0.22W/m^0C)(0.015x1m^2)} = 48.48\,^0C/W$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.16m}{(0.72W/m^0C)(0.22x1m^2)} = 1.01\,^0C/W$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25W/m^{2\,0}C)(0.25x1m^2)} = 0.16^0C/W$$

solution

The three resistance R_3, R_4 and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 W / °C$$

Which give

$$R_{mid} = 0.97 °C / W$$

Now all the resistance are in series, and the total resistance is

$$R_{mid} = R_i + R_1 + R_2 + R_{mid} + R_6 + R_0$$
$$= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 = 6.85 °C / W$$

solution

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (10)]°C}{6.85°C / W} = 4.38 W \quad (\text{per } 0.25\ \text{m}^2 \text{surface area})$$

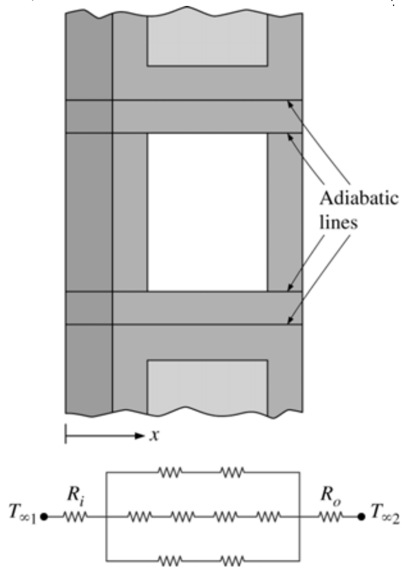
or $4.38/0.25 = 17.5$ W per m^2 area. The total area of the wall is

$A = 3\text{m} \times 5\text{m} = 15\text{m}^2$.

Then the rate of heat transfer throuugh the entire wall becomes

$$\dot{Q}_{tatal} = (17.5 W / m^2)(15 m^2) = 263 W$$

FIGURE 3–22



Alternative thermal resistance network for example 3-6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

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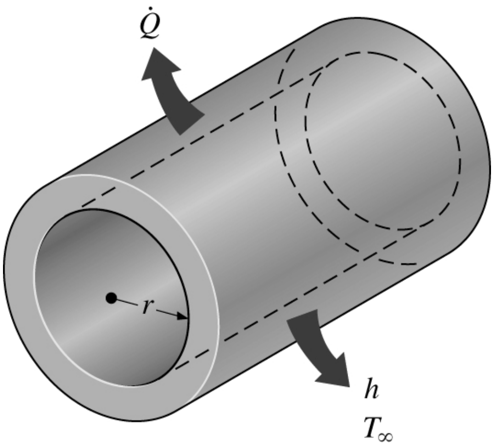
FIGURE 3–23



Heat is lost from a hot water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

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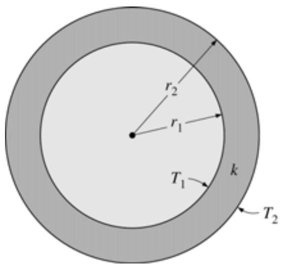
FIGURE 3–24



$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr}$$

$$\int_{r=r_1}^{r=r_2} \frac{\dot{Q}_{cond,cyl}}{A} dr = -\int_{T=T_1}^{T=T_2} kdT \rightarrow A = 2\pi rL$$

$$\dot{Q}_{cond,cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

$R_{cyl}=\ln(r_2/r_1)/2\pi Lk$

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

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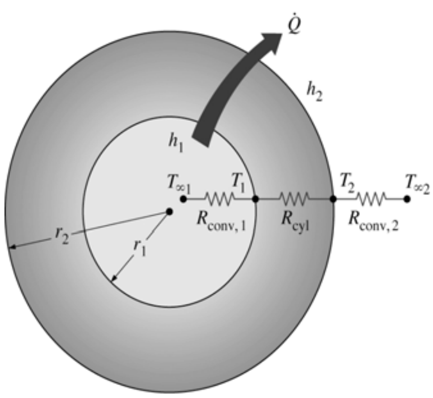
FIGURE 3–25



$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$

The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

40

FIGURE 3–25

The rate of heat transfer under steady condition can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

where $R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$

$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

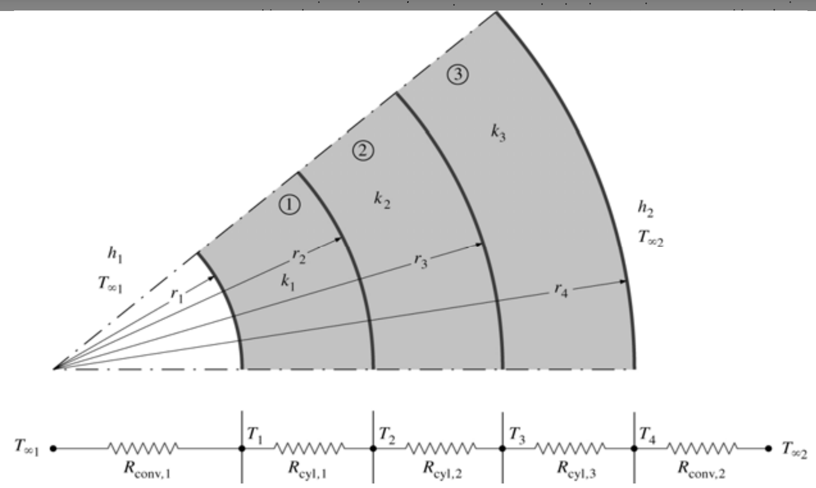
where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{cyl,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

We could also calculate T_2 from

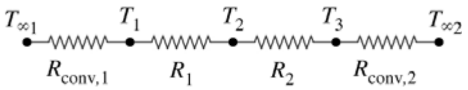
$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv,2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$

FIGURE 3–26



The thermal resistance network for heat transfer through a three-layered composite cylinder

FIGURE 3–27



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$$
$$= \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1}$$
$$= \frac{T_1 - T_3}{R_1 + R_2}$$
$$= \frac{T_2 - T_3}{R_2}$$
$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{conv,2}}$$
$$= \cdots$$

The ratio ΔT/R across any layer is equal to $\dot{Q}$ Which remains constant in one-dimensional steady conduction.

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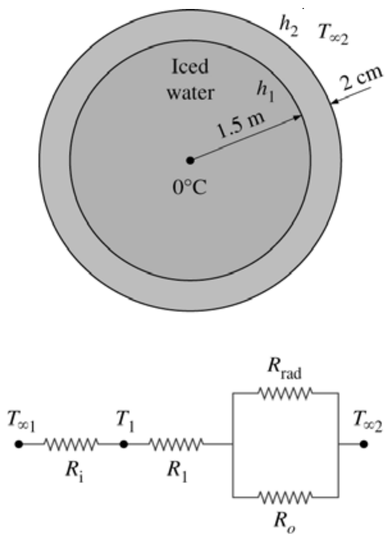
FIGURE 3–28



Schematic for Example 3-7.

Assumptions
1. steady state heat transfer
2. (1-D) heat transfer and thermal symmetry at midpoint
3. thermal conductivity is constant

Properties
1. the thermal conductivity of steel is given to be k=15W/m*C
2. Heat of fusion of water at atmospheric is h_{if}=333.7 kj/kg
3. Emissivity outer surface of tank is ε=1

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solution (a)

Inner and outer surface area of the tank are

$$A_1 = \pi D_1^2 = \pi(3)^2 = 28.3m^2$$
$$A_2 = \pi D_2^2 = \pi(3.04)^2 = 29.0m^2$$

Radiation heat transfer coefficient is given by

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$
$$h_{rad} = (1)(5.67*10^{-8})\left[(295)^2 + (278)^2\right]\left[295 + 278\right] = 5.34 \quad W/m^{2\,0}C$$

Then the individual thermal resistance become

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(80W/m^{2\,0}C)(28.3m^2)} = 0.000442 \quad {}^0C/W$$
$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{1.52 - 1.5}{4\pi(15W/m^0C)(1.52m)(1.50m)} = 0.0.000047 \ {}^0C/W$$
$$R_o = R_{conv2} = \frac{1}{h_2 A_2} = \frac{1}{(10W/m^{2\,0}C)(29.0m^2)} = 0.00345 \quad {}^0C/W$$
$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{(5.34W/m^{2\,0}C)(29.0m^2)} = 0.00646 \quad {}^0C/W$$
$$R_{total} = R_i + R_1 + \left(\frac{1}{R_o} + \frac{1}{R_{rad}}\right)^{-1} = 0.000442 + 0.000047 + \left(\frac{1}{0.00345} + \frac{1}{0.00646}\right)^{-1} = 0.00274\,{}^0C/W$$

Then Steady rate of heat transfer through the window become

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{[22-0]}{0.00274} = 8029 \quad W \; or \; \dot{Q} = 8.027 kJ/s$$

We now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}}$$

Which give

$$R_{equiv} = \left(\frac{1}{R_o} + \frac{1}{R_{rad}}\right)^{-1} = \left(\frac{1}{0.00345} + \frac{1}{0.00646}\right)^{-1} = 0.00225$$

$$T_2 = T_{\infty 2} - \dot{Q}R_{equiv} = 22 - (8029)(0.00225) = 4^0C$$

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Solution (b)

The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\Delta t = (8.029 kJ/s)(24x3600s) = 673{,}700 \quad kJ$$

Note that it take 333.7kJ of energy to melt 1 kg of ice at 0 °C , the amount of ice that will melt during a 24-h period is

$$m_{ice} = \frac{Q}{h_{if}} = \frac{673{,}700 \; kJ}{333.7 \, kJ \; / \; kg} = 2079 \; kg$$

Discussion

$$R_{combined} = \frac{1}{h_{combined} A_2} = \frac{1}{(15.34W/m^2.°C)(29m^2)} = 0.00225^0 C/W$$

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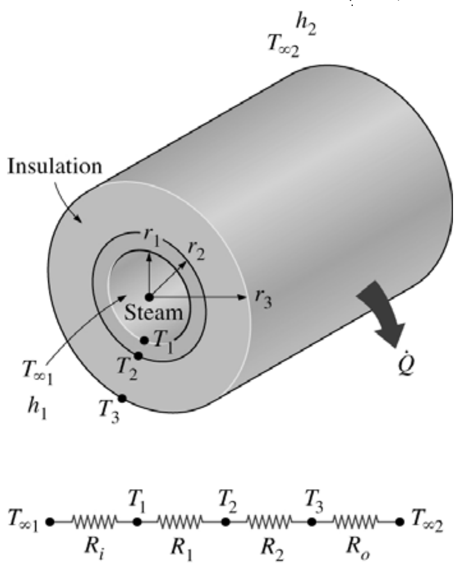
FIGURE 3–29



Schematic for Example 3-8.

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<u>Assumptions</u>

1. steady state heat transfer
2. (1-D) heat transfer and
   thermal symmetry at centerline and variation in the axial direction
3. thermal conductivity is constant
4. thermal contact resistance at interface is negligible

<u>Properties</u>

1.the thermal conductivity of cast iron is given to be k =80W/m*C and k = 0.05W/m*C for glass wool insulation

50

The area of the surface exposed to convection are determine to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025m)(1m) = 0.157m^2$$
$$A_2 = 2\pi r_2 L = 2\pi(0.0575m)(1m) = 0.361m^2$$

Then the individual thermal resistance become

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(60W/m^{2\,0}C)(0.157m^2)} = 0.106\ {}^0C/W$$
$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80W/m^0C)(1m)} = 0.0002\ {}^0C/W$$
$$R_2 = R_{insulation} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05W/m^0C)(1m)} = 2.35\ {}^0C/W$$

$$R_o = R_{conv2} = \frac{1}{h_2 A_3} = \frac{1}{(18W/m^{2\,0}C)(0.361m^2)} = 0.154\ {}^0C/W$$
$$R_{total} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^0C/W$$

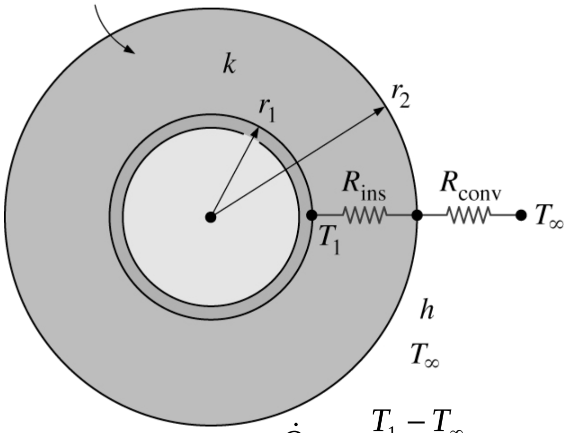
Then Steady rate of heat loss from the steam become

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[320-5]}{2.16} = 121\ W$$

Temperature drop across the pipe and the insulation are determine

$$\Delta T_{pipe} = \dot{Q}R_{pipe} = (121W)(0.0002^0C/W) = 0.02^0C$$
$$\Delta T_{insulation} = \dot{Q}R_{insulation} = (121W)(2.35^0C/W) = 284^0C$$

FIGURE 3–30



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$

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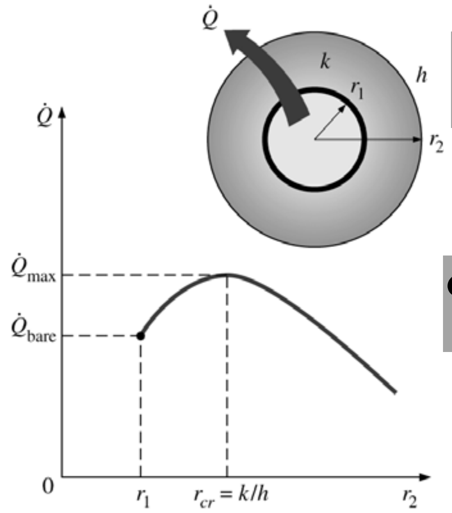
FIGURE 3–31



Critical radius of insulation for a cylindrical body to be

$$r_{cr,cylinder} = \frac{k}{h} \quad (m)$$

Critical radius of insulation for a spherical shell is

$$r_{cr,sphere} = \frac{2k}{h} \quad (m)$$

where k is the thermal conductivity

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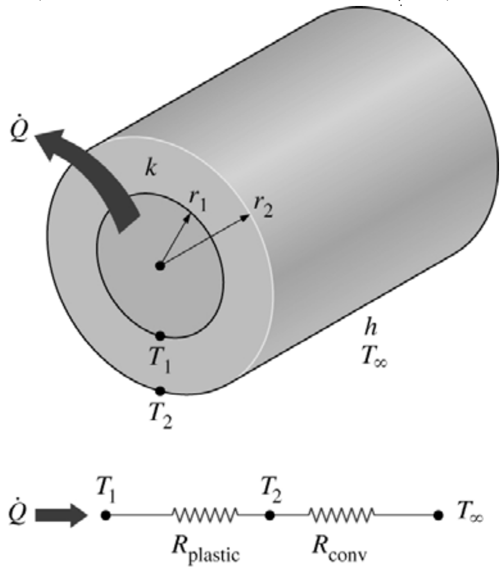
FIGURE 3–32



Schematic for Example 3-9.

Assumptions

1. steady state heat transfer
2. (1-D) heat transfer and thermal symmetry at centerline and variation in the axial direction
3. thermal conductivity is constant
4. thermal contact resistance at interface is negligible
5. heat transfer coefficient incorporates the radiation effect

Properties

1.the thermal conductivity of plastic is given to be k=0.15W/m*C

The rate of heat transfer become equal to the heat generate within the wire, which is determine to be

$$\dot{Q} = \dot{W}_e = VI = (8V)(10\,A) = 80\,W$$

The values of these two resistances are determine to be

$$A_2 = 2\pi r_2 L = 2\pi(0.0035m)(5m) = 0.110m^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12W/m^{2\,0}C)(0.110m^2)} = 0.76\ {}^0C/W$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15W/m^0C)(5m)} = 0.18\,{}^0C/W$$

therefore

$$R_{total} = R_{conv} + R_{plastic} = 0.76 + 0.18 = 0.94^0C/W$$

We now determine the interface temperature from

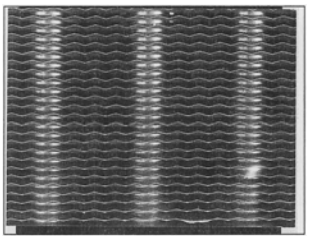
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \Rightarrow T_1 = T_\infty + \dot{Q}R_{total}$$

$$= 30 + (80W)(0.94^0C/W) = 105^0C$$

The critical radius of insulation of the plastic cover is determine from

$$r_{cr} = \frac{k}{h} = \frac{0.15W/m^0C}{12W/m^{2\,0}C} = 0.0125m$$

FIGURE 3–33



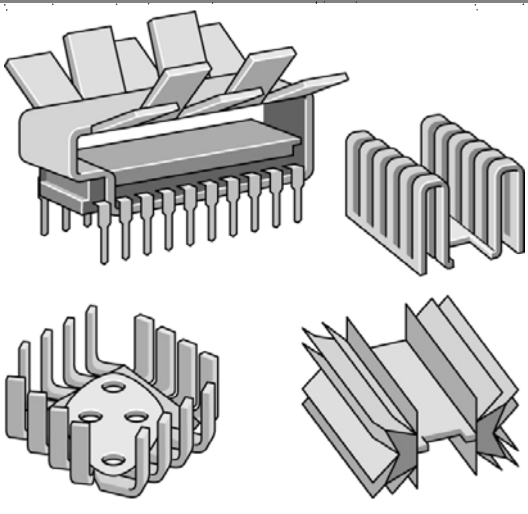
HEAT TRANSFER FROM FINNED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$\dot{Q}_{conv} = hA_S(T_S - T_\infty)$$

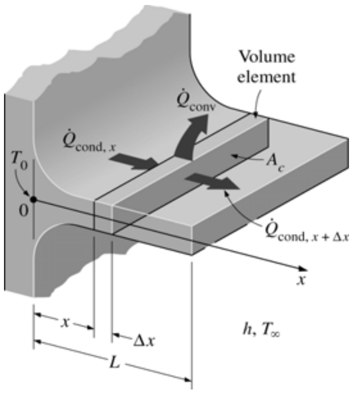
The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air (photo by Yunus Cengel and James Kleiser).

FIGURE 3–34



Some innovative fin designs.

FIGURE 3–35



Fin Equation

The energy balance on this volume element can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction in.to}\\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of heat}\\ \text{conduction from the}\\ \text{element at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{convection from}\\ \text{the element}\end{pmatrix}$$

$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$$

where

$$\dot{Q}_{conv} = h(p\Delta x)(T - T_\infty)$$

Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

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Δx

FIGURE 3–35

Fin Equation

Substituting and dividing by Δx **,we obtain**

$$\frac{\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x}}{\Delta x} + hp(T - T_\infty) = 0$$

Taking the limit as $\Delta x \to 0$ **gives**

$$\frac{d\dot{Q}_{cond}}{dx} + hp(T - T_\infty) = 0$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$$

Gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c \frac{dt}{dx}\right) - hp(T - T_\infty) = 0$$

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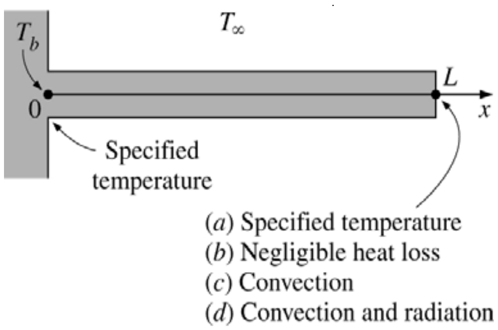
FIGURE 3–36



Boundary conditions at the fin base

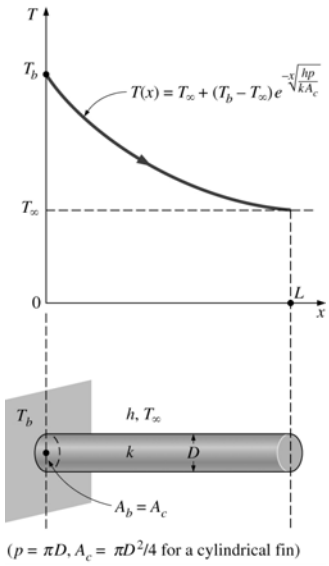
$$\theta(0) = \theta_b = T_b - T_\infty$$

Boundary conditions at the fin tip.

$$\theta(L) = T(L) - T_\infty = 0 \quad ; L \to \infty$$

Boundary conditions at the fin base and the fin tip.

FIGURE 3–37



Very long fin:

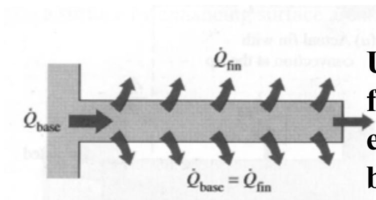
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}}$$

$$\dot{Q}_{long\ fin} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

$$\dot{Q}_{fin} = \int_{A_{fin}} h[T(x) - T_\infty]dA_{fin} = \int_{A_{fin}} h\theta(x)dA_{fin}$$

A long circular fin of uniform cross section and the variation of temperature along it.

FIGURE 3–38



Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

Boundary condition at fin tip: $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

Adiabatic fin tip:

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh a(L-x)}{\cosh aL}$$

$$\dot{Q}_{insulated\ tip} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

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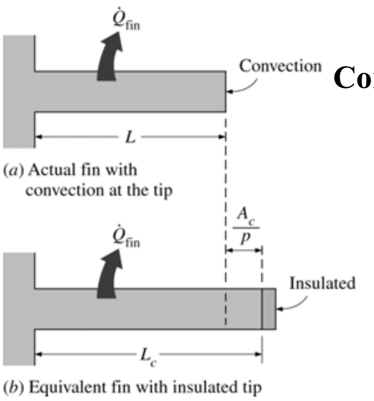
FIGURE 3–39



Corrected fin length: $L_c = L + \frac{A_c}{p}$

$$L_{c,rectangular\ fin} = L + \frac{t}{2}$$

and

$$L_{c,cylinder\ fin} = L + \frac{D}{4}$$

Corrected fin length L_c is defined such that heat transfer from fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip

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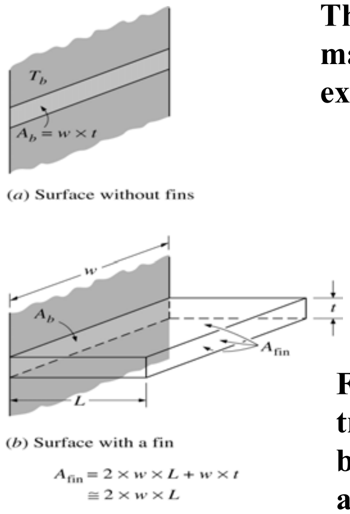
FIGURE 3–40

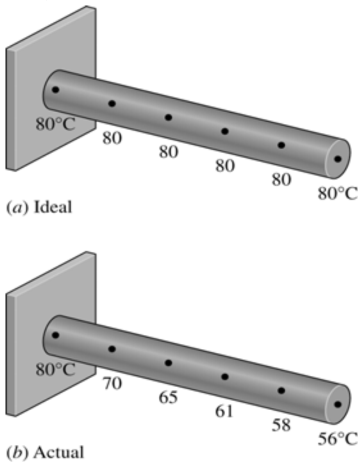


The heat transfer from the fin will be maximum in this case and can be expressed as

$$\dot{Q}_{fin,\max} = hA_{fin}\left(T_b - T_\infty\right)$$

Fins enhance heat transfer from a surface by enhancing surface area.

FIGURE 3–41



Fin efficiency

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{Actual\,heat\,transfer\,rate\,\,from\,the\,\,fin}{Ideal\,heat\,transfer\,rate\,\,from\,the\,\,fin}$$

or

$$\dot{Q}_{fin} = \eta_{fin}\dot{Q}_{fin,\max} = \eta_{fin} hA_{fin}\left(T_b - T_\infty\right)$$

Temperature distribution in a fin.

FIGURE 3–41

Fin efficiency

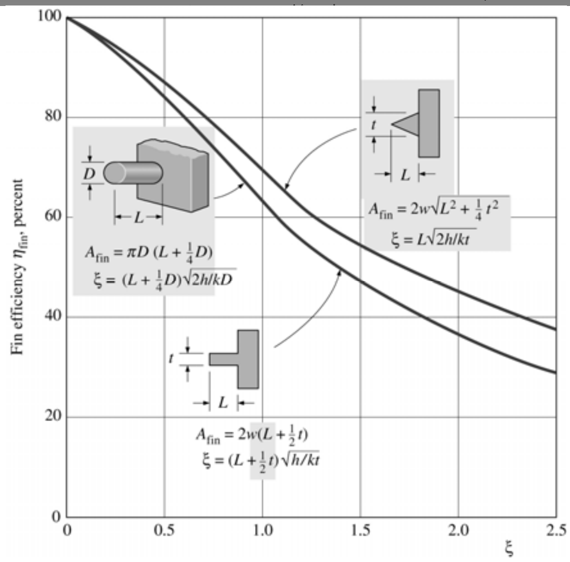
For the cases of constant cross section of very long fins and fins with insulated tips, the fin efficient can be expressed as

$$\eta_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)}{hA_{fin}\left(T_b - T_\infty\right)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL}$$

and

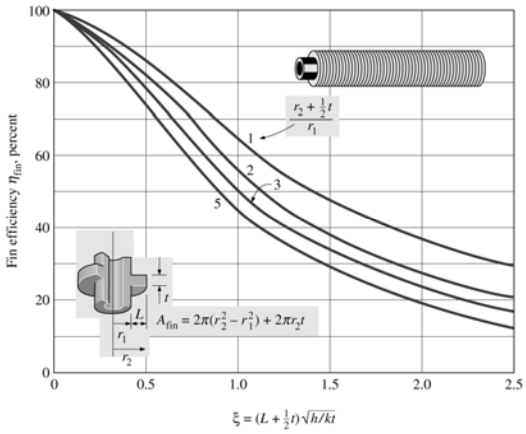
$$\eta_{insulated\ \text{tip}} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)\tanh aL}{hA_{fin}\left(T_b - T_\infty\right)} = \frac{\tanh aL}{aL}$$

FIGURE 3–42



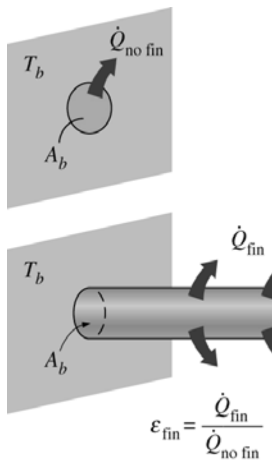
Efficiency of circular, rectangular, and triangular fins on a plain surface of width w

FIGURE 3–43



Efficiency of circular fin of length L and constant thickness t.

FIGURE 3–44



Fin Effectiveness

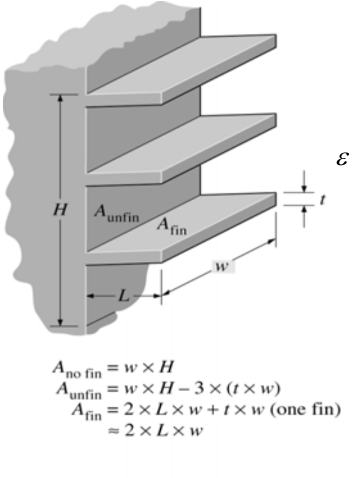
$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of base area } A_b}{\text{Heat transfer rate from the surface area } A_b}$$

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)} = \frac{\eta_{fin} hA_{fin}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{fin}}{A_b}\eta_{fin}$$

The effectiveness of such a long fin is determined to be

$$\varepsilon_{long\ fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

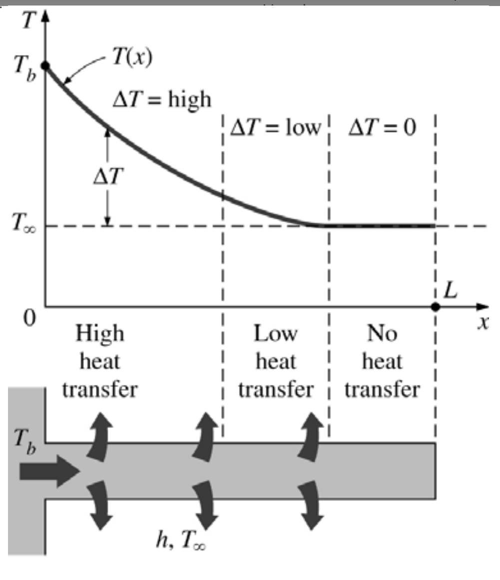
FIGURE 3–45



$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,nofin}} = \frac{h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)}{hA_{no,fin}(T_b - T_\infty)}$$

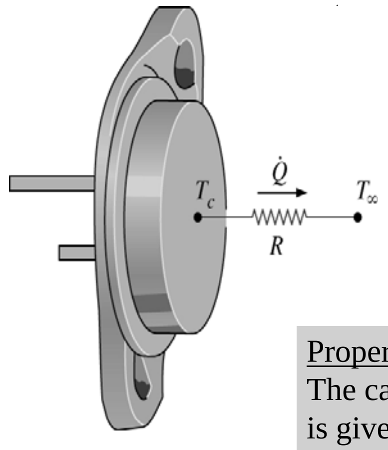
Various surface areas associated with a rectangular surface with three fins.

FIGURE 3–46



Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

FIGURE 3–47



Schematic for Example 3-10.

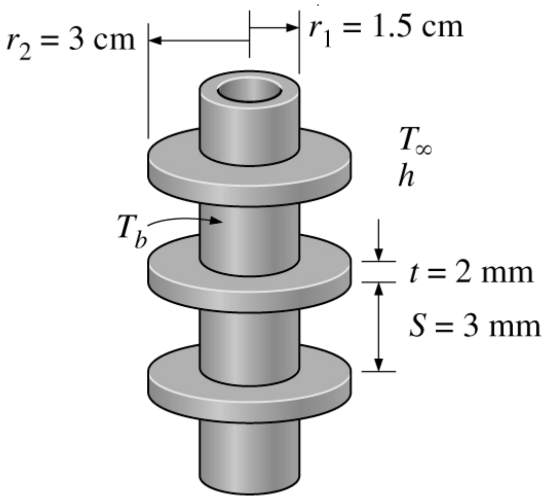
Assumptions
1. Steady operating conditions exist
2. The transistor case is isothermal at $85°C$

Properties

The case-to-ambient thermal resistance is given to be $20°C/W$

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{case-ambient} = \frac{T_c - T_\infty}{R_{case-ambient}} = \frac{(85-25)°C}{20°C/W} = 3W$$

FIGURE 3–48



Schematic for Example 3-12.

Assumptions

1. steady operating conditions exist
2. The heat transfer coefficient is uniform over the entire fin surfaces
3. thermal conductivity is constant
4. heat transfer by radiation is negligible

Properties

1.the thermal conductivity of the fin is given to be k=180W/m*C

In the case of no fin, heat transfer from the tube per mater of its length is determined from Newton's law of cooling to be

$$A_{no\,fin} = \pi D_1 L = \pi(0.03m)(1m) = 0.0942m^2$$

$$\dot{Q}_{no\,fin} = hA_{no\,fin}(T_b - T_\infty) = (60)(0.0942)(120-25) = 537W$$

The efficiency of the circular fins attached to a circular tube is plotted in fig.3-43 Noting that $L=1/2(D_2-D_1)=1/2(0.06-0.03)=0.015m$ in this case we have

$$\frac{r_2+\frac{1}{2}t}{r_1} = \frac{(0.03+\frac{1}{2}*0.002)}{0.015} = 2.07 \quad \textbf{(1)}$$

$$(L+\frac{1}{2}t)\sqrt{\frac{h}{kt}} = (0.015+\frac{1}{2}*0.002)*\sqrt{\frac{60W/m^{2\,0}C}{(180W/m^0C)(0.002m)}} = 0.207 \quad \textbf{(2)}$$

From (1),(2) → $\eta_{fin} = 0.95$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$
$$= 2\pi\left[(0.03m)^2 - (0.015m)^2\right] + 2\pi(0.03m)(0.002m)$$
$$= 0.00462m^2$$

$$\dot{Q}_{fin} = \eta_{fin}\dot{Q}_{fin,\max} = \eta_{fin} h A_{fin}(T_b - T_\infty)$$
$$= 0.95(60W/m^{2\,0}C)(0.00462m^2)(120-25)^0C$$
$$= 25.0W$$

Heat transfer from the unfinned portion of the tube is

$$A_{unfin} = \pi D_1 S = \pi(0.03m)(0.003m) = 0.000283m^2$$
$$\dot{Q}_{unfin} = hA_{unfin}(T_b - T_\infty)$$
$$= (60W/m^{2\,0}C)(0.000283m^2)(120-25)^0C = 1.60W$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{total,fin} = n(\dot{Q}_{fin} + \dot{Q}_{unfin}) = 200(25.0 + 1.6) = 5320W$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{increase} = \dot{Q}_{total,fin} - \dot{Q}_{no,fin} = 5320 - 537 = 4783W \quad (per\ m\ tube\ length)$$

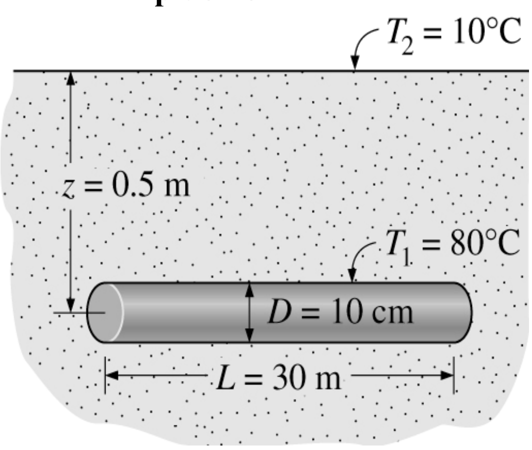
Discussion **The overall effectiveness of the finned tube is**

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,no\ fin}} = \frac{5320\ W}{537\ W} = 9.9$$

81

FIGURE 3–49

Schematic for Example 3-13.



The shape factor for this configuration

$$S = \frac{2\pi L}{\ln(4z/D)}$$

$$S = \frac{2\pi x 30m}{\ln(4x0.5/0.1)} = 62.9m$$

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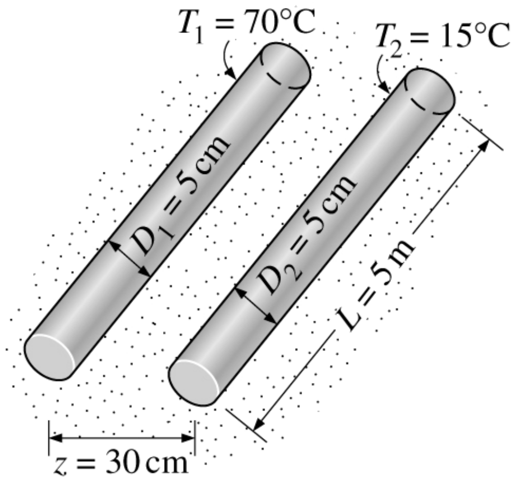
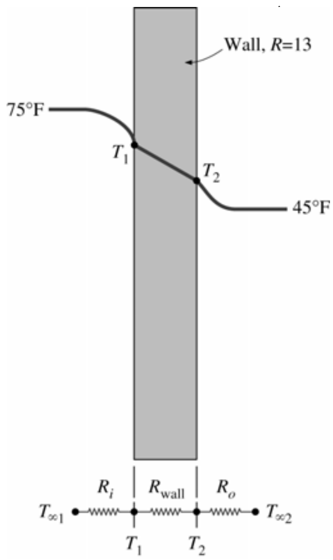
FIGURE 3–50



Schematic for Example 3-14.

83

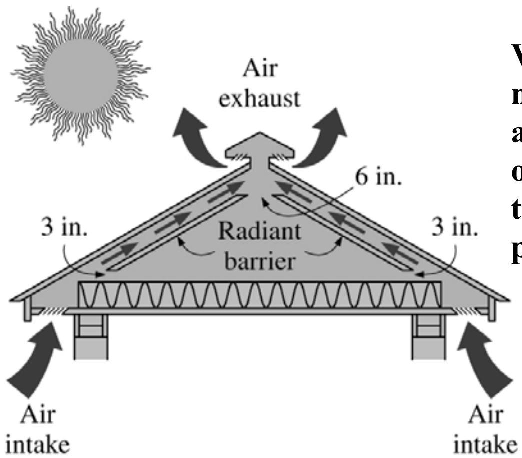
FIGURE 3–51



Schematic for Example 3-15.

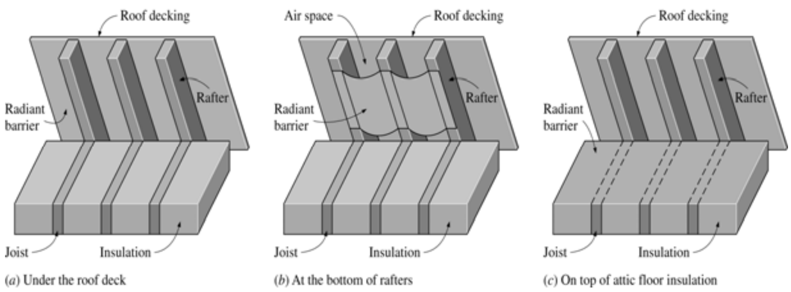
84

FIGURE 3–52



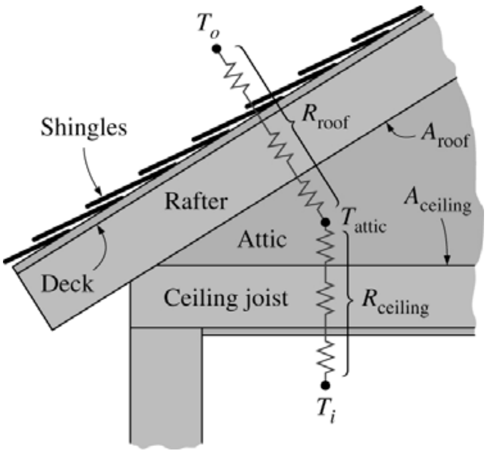
Ventilation paths for a naturally ventilated attic and the appropriate size of the flow area around the radiant barrier for proper air circulation.

FIGURE 3–53



Three possible locations for an attic radiant barrier.

FIGURE 3-54



Thermal resistance network for a pitched roof-attic-ceiling combination for the case of an unvented attic.

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