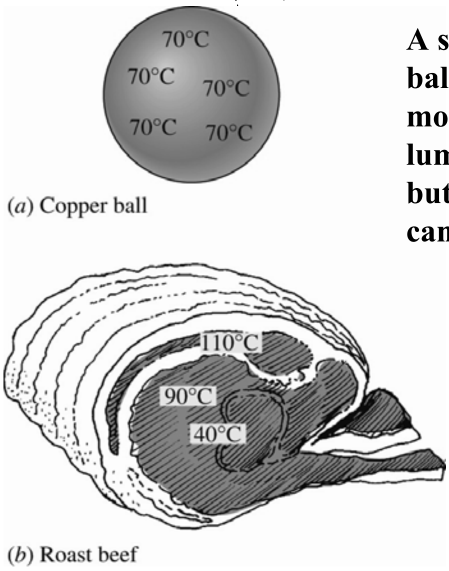
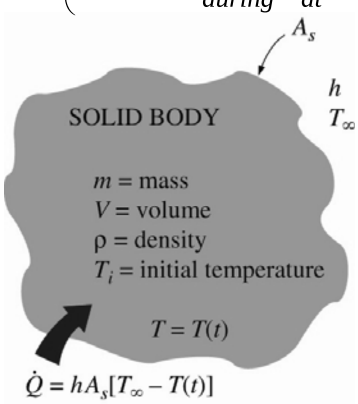
FIGURE 4-1



A small copper ball can be modeled as a lumped system, but a roast beef cannot

FIGURE 4-2

$$\begin{pmatrix} \textit{Heat transfer into the body} \\ \textit{during } dt \end{pmatrix} = \begin{pmatrix} \textit{The increase in the} \\ \textit{energy of the body} \\ \textit{during } dt \end{pmatrix}$$



$$hA_s(T_\infty - T)dt = mC_p dT$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V C_p} dt$$

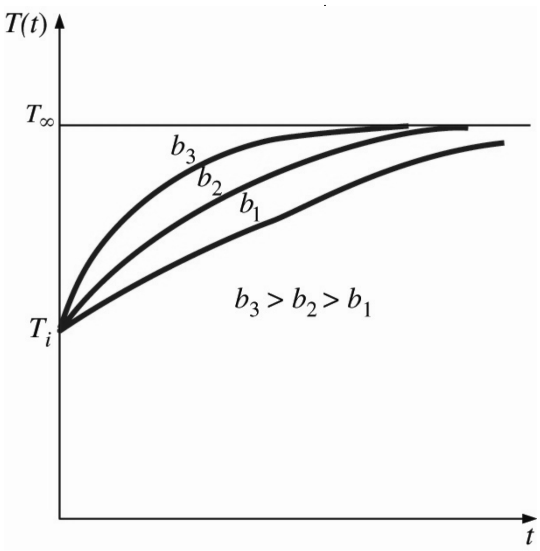
$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V C_p} t$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V C_p}$$

The geometry and parameters involved in the lumped system analysis.

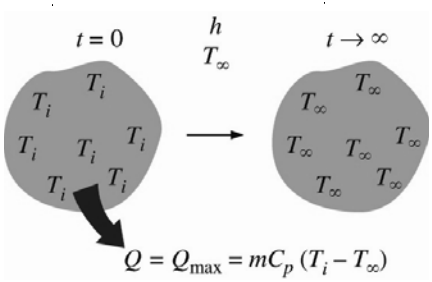
FIGURE 4-3



The temperature of a lumped system approaches the environment temperature as time gets larger.

3

FIGURE 4-4



$$\dot{Q}(t) = hA_s\left[T(t) - T_\infty\right]$$

$$Q = mC_p\left[T(t) - T_i\right]$$

$$Q_{\max} = mC_p\left(T_\infty - T_i\right)$$

$$L_c = \frac{V}{A_s}$$

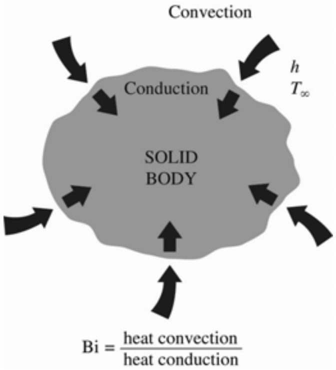
$$Bi = \frac{hL_c}{k}$$

Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

4

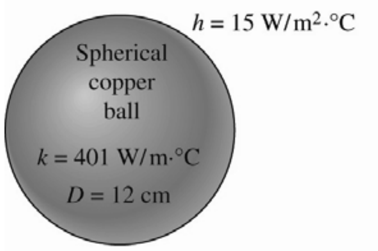
FIGURE 4-5

$$Bi = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{Convection \ \ at \ \ the \ \ surface \ \ of \ \ the \ \ body}{Conduction \ \ within \ \ the \ \ body}$$



$$Bi = \frac{L_c/k}{1/h} = \frac{Conduction \ resistance \ \ within \ \ the \ \ body}{Convection \ resistance \ \ at \ \ the \ \ surface \ \ of \ \ the \ \ body}$$

FIGURE 4-6

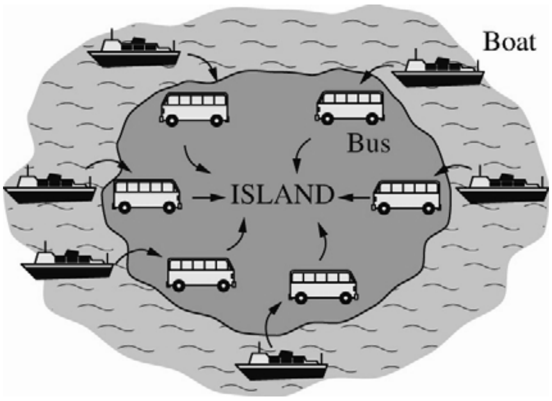


$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion lumped system analysis.

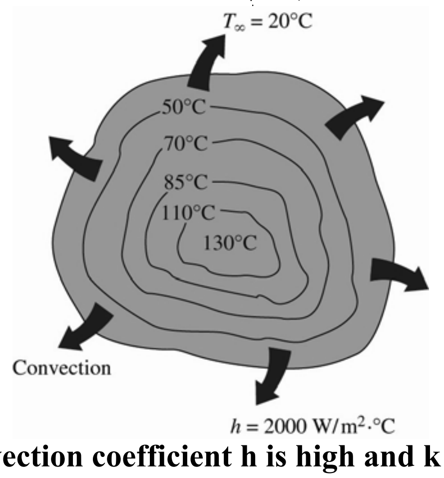
FIGURE 4-7



Analogy between heat transfer to a solid and passenger traffic to an island.

7

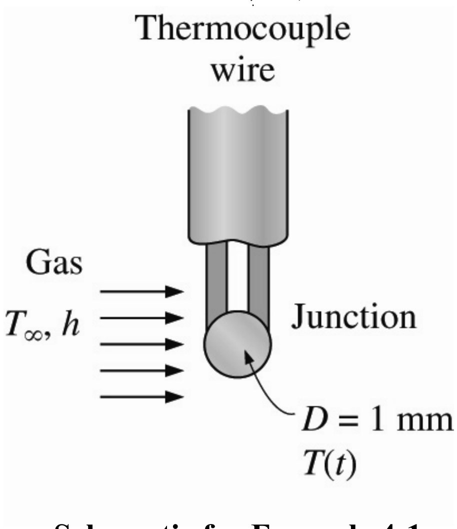
FIGURE 4-8



When the convection coefficient h is high and k is low, large temperature difference occur between the inner and outer regions of a large solid.

8

FIGURE 4-9



Schematic for Example 4-1.

9

Assumption

- The junction is spherical in shape with a diameter of D = 0.001 m.
- Thermal properties of the junction and the heat transfer coefficient are constant.
- Radiation effects are negligible

10

Analysis

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = \frac{1}{6}(0.001\,m) = 1.67 \times 10^{-4}\,m$$

$$Bi = \frac{hL_c}{k} = \frac{(210 W/m^2 \bullet {}^O C)(1.67 \times 10^{-4} m)}{35 W/m \bullet {}^O C} = 0.001 < 0.1$$

In order to read 99 percent of the initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

Analysis

The value of the exponent b is

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{210 W/m^2 \bullet {}^O C}{(8500\,kg/m^3)(320\,J/kg \bullet {}^O C)(1.67 \times 10^{-4}\,m)} = 0.462\,s^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \rightarrow 0.01 = e^{-(0.462\,s^{-1})t}$$

$$\Rightarrow \quad t = 10\,s$$

FIGURE 4-10

Schematic for Example 4-2.

Assumption

.The body can be modeled as a 30-cm-diameter, 1.70-m-long cylinder.
.The thermal properties of the body and the heat transfer coefficient are constant.
.The radiation effects are negligible.
.The person was healthy (!) when he or she died with a body temperature of 37 ^{0}C.

Analysis

$$L_c = \frac{V}{A_s} = \frac{\pi r_0^2 L}{2\pi r_0 L + 2\pi r_0^2} = \frac{\pi (0.15m)^2 (1.7m)}{2\pi (0.15m)(1.7m) + 2\pi (0.15m)^2} = 0.0689m$$

$$Bi = \frac{hL_c}{k} = \frac{(8W/m^2 \bullet {}^O C)(0.0689m)}{0.617W/m \bullet {}^O C} = 0.89 > 0.1$$

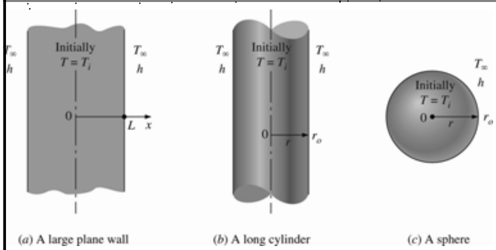
$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{8W/m^2 \bullet {}^O C}{(996kg/m^3)(4178J/kg \bullet {}^O C)(0.0689m)} = 2.79 \times 10^{-5} s^{-1}$$

Analysis

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \rightarrow \frac{25 - 20}{37 - 20} = e^{-(2.79\times 10^{-5} s^{-1})t}$$

$$t = 43{,}860 s = 12.2 h$$

FIGURE 4-11



Schematic of the simple geometries in which heat transfer is one-dimensional.

Dimensionless time

$$\tau = \frac{\alpha t}{L^2}$$

Dimensionless temperature

$$\theta(x,t) = \frac{T(x,t) - T_\infty}{T_i - T_\infty}$$

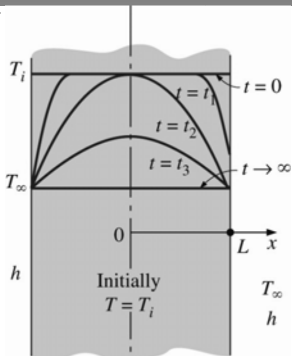
Dimensionless distance from the center

$$X = \frac{x}{L}$$

Dimensionless heat transfer coefficient

$$Bi = \frac{hL}{k}$$

FIGURE 4-12



Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.

Plane wall

$$\theta(x,t)_{wall} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2$$

Cylinder

$$\theta(r,t)_{cyl} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_0), \quad \tau > 0.2$$

Sphere

$$\theta(r,t)_{sph} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0}, \quad \tau > 0.2$$

Center of plane wall (x=0)

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

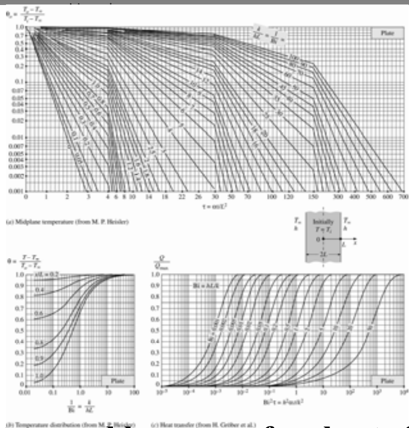
Center of cylinder (r=0)

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Center of sphere (r=0)

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

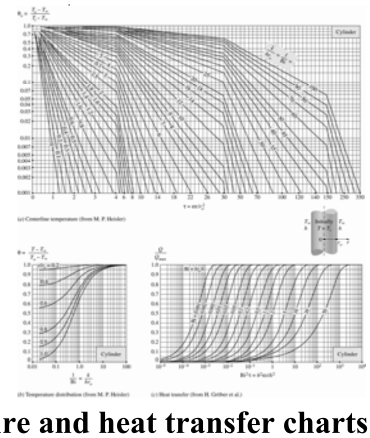
FIGURE 4-13



Transient temperature and heat transfer charts for a plane wall of thickness 2L initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

19

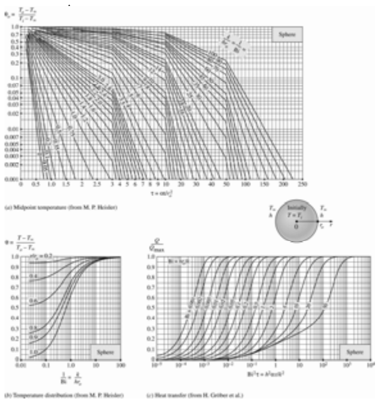
FIGURE 4-14



Transient temperature and heat transfer charts for a long cylinder of radius r_0 initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

20

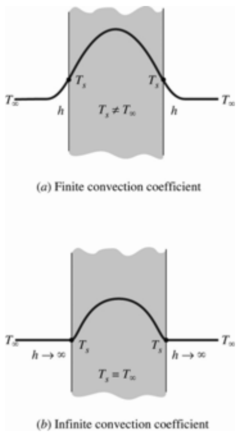
FIGURE 4-15



Transient temperature and heat transfer charts for a sphere of radius r_0 initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

21

FIGURE 4-16

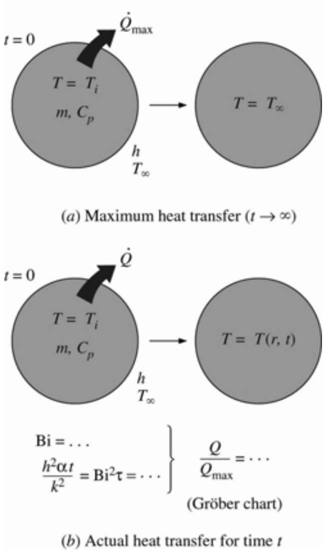


$$Q_{\max} = mC_p(T_\infty - T_i)$$
$$= \rho V C_p(T_\infty - T_i)$$

The specified surface temperature corresponds to the case of convection to an environment at T_∞ with a convection coefficient h that is infinite.

22

FIGURE 4-17



The fraction of total heat transfer Q/Q_{max} up to a specified time t is determined using the Gröber charts.

Plane wall

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{0,wall}\frac{\sin\lambda_1}{\lambda_1}$$

Cylinder

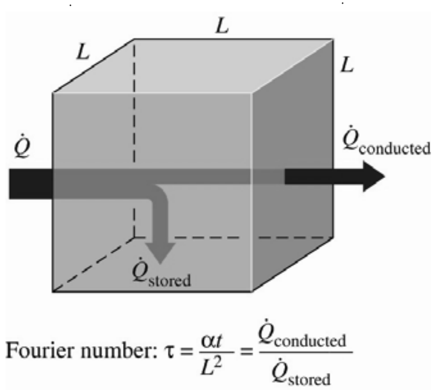
$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{0,cyl}\frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere

$$\left(\frac{Q}{Q_{\max}}\right)_{sph} = 1 - 3\theta_{0,sph}\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3}$$

23

FIGURE 4-18

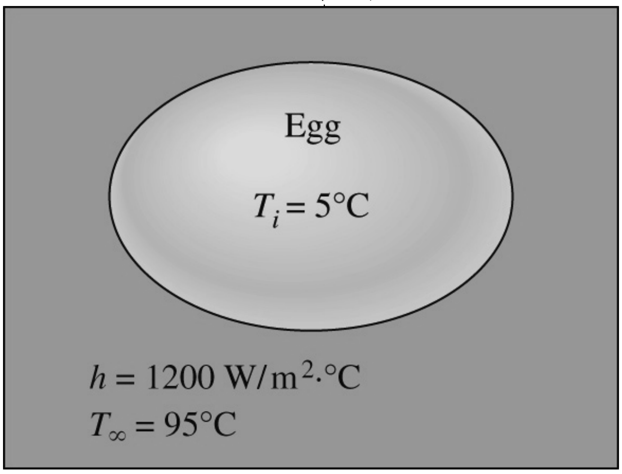


Fourier number at time t can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2(1/L)}{\rho C_p L^3/t}\frac{\Delta T}{\Delta T} = \frac{\textit{The rate at which heat is conducted across } L \textit{ of a body of volume } L^3}{\textit{The rate at which heat is stored in a body of vulume } L^3}.$$

24

FIGURE 4-19



Schematic for Example 4-3.

Assumption

- .The egg is spherical in shape with a radius of r_0=2.5cm.
- .Heat conduction in the egg is one-dimension because of thermal symmetry about the midpoint.
- .The thermal properties of egg and the heat transfer coefficient are constant.
- The Fourier number is τ>0.2 so that the one term approximate solutions are applicable.

Analysis

$$Bi = \frac{hr_0}{k} = \frac{(1200W/m^2 \bullet {}^{O}C)(0.025m)}{0.627W/m \bullet {}^{O}C} = 47.8 > 0.1$$

The coefficients λ_1 and A_1 for a sphere corresponding to this Bi are, from table 4-1,

λ_1=3.0753

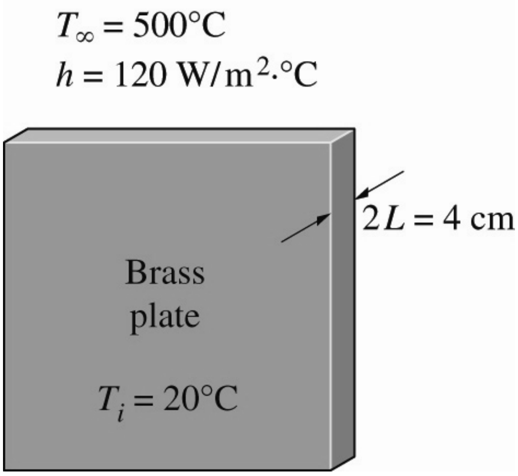
A_1=1.9958

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{70-95}{5-95} = 1.9958 e^{-(3.0753)^2 \tau} \rightarrow \tau = 0.209 > 0.2$$

Analysis

$$t = \frac{\tau r_0^2}{\alpha} = \frac{(0.209)(0.025m)^2}{0.151 \times 10^{-6} m^2/s} = 865s \approx 14.4 \min$$

FIGURE 4-20



Schematic for Example 4-4.

29

Assumption

•. Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane.
•The thermal properties of the plate and the heat transfer coefficient are constant.
•The Fourier number is τ>0.2 so that the one-term approximate solutions are applicable.

30

Analysis

$$\frac{1}{Bi} = \frac{k}{hL} = \frac{100\,W/m\bullet^{o}C}{\left(120\,W/m^{2}\bullet^{o}C\right)\left(0.02m\right)} = 45.8$$

$$\tau = \frac{\alpha t}{L^2} = \frac{\left(33.9\times10^{-6}m^{2}/s\right)\left(7\times60s\right)}{\left(0.02m\right)^{2}} = 35.6$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.46$$

$$\frac{1}{Bi} = \frac{k}{hL} = 45.8$$

Analysis

$$\frac{x}{L} = \frac{L}{L} = 1$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.99$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{T - T_\infty}{T_0 - T_\infty}\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.46\times0.99 = 0.455$$

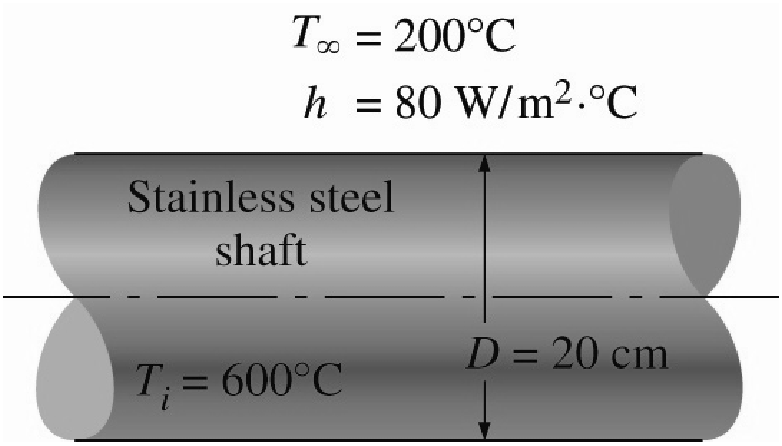
$$b = \frac{hA_s}{\rho C_p V} = \frac{h(2A)}{\rho C_p (2LA)} = \frac{h}{\rho C_p L} = \frac{120W/m^2\bullet^{o}C}{\left(8530kg/m^3\right)\left(380J/kg\bullet^{o}C\right)\left(0.02m\right)} = 0.00185s^{-1}$$

Analysis

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt}\rightarrow\frac{T(t)-500}{20-500}=e^{-(0.00185\,s^{-1})(420\,s)}$$

$$\Rightarrow\quad T(t)=279\,^oC$$

FIGURE 4-21



Schematic for Example 4-5.

Assumption

•Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the centerline.
•The thermal properties of the shaft and the heat transfer coefficient are constant.
•The Fourier number is τ>0.2 so that the one-term approximate solutions are applicable.

Analysis

$$\frac{1}{Bi} = \frac{k}{hr_0} = \frac{14.9 W/m \bullet {}^oC}{(80 W/m^2 \bullet {}^oC)(0.1 m)} = 1.86$$

$$\tau = \frac{\alpha t}{r_0^2} = \frac{(3.95 \times 10^{-6} m^2/s)(45 \times 60 s)}{(0.1 m)^2} = 1.07$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.40$$

$$T_0 = T_\infty + 0.4(T_i - T_\infty) = 200 + 0.4(600 - 200) = 360\,{}^oC$$

Analysis

$$m = \rho V = \rho \pi r_0^2 L = (7900\, kg/m^3)\pi (0.1m)^2 (1m) = 248.2kg$$

$$Q_{\max} = mC_p (T_\infty - T_i) = (248.2kg)(0.477\, kJ/kg \bullet {}^oC)(600 - 200)^oC = 47{,}354\, kJ$$

$$Bi = \frac{1}{1/Bi} = \frac{1}{1.86} = 0.537$$

$$\frac{h^2 \alpha t}{k^2} = Bi^2 \tau = (0.537)^2 (1.07) = 0.309$$

$$\frac{Q}{Q_{\max}} = 0.62$$

$$Q = 0.62 Q_{\max} = 0.62 \times (47{,}354\, kJ) = 29{,}360\, kJ$$

Analysis Alternative solution

$$Bi = \frac{hr_0}{k} = \frac{(80W/m^2 \bullet {}^OC)(0.1m)}{14.9W/m \bullet {}^OC} = 0.537$$

$$\lambda_1 = 0.970 \qquad A_1 = 1.122$$

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.122 e^{-(0.970)^2 (1.07)} = 0.41$$

$$T_0 = T_\infty + 0.41(T_i - T_\infty) = 200 + 0.41(600 - 200) = 364\,{}^oC$$

Analysis Alternative solution

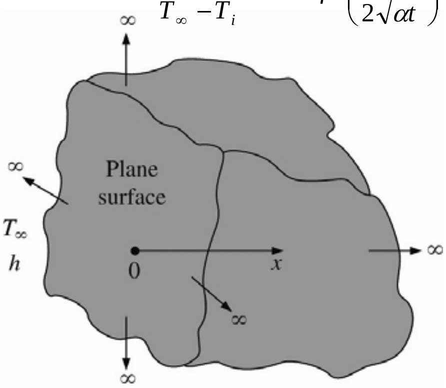
$$\frac{Q}{Q_{\max}} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{(\lambda_1)} = 1 - 2 \times 0.41 \frac{0.430}{0.970} = 0.636$$

$$Q = 0.636 Q_{\max} = 0.636 \times (47{,}354\, kJ) = 30{,}120\, kJ$$

FIGURE 4-22

$$1 - \theta(x,t) = 1 - \frac{T(x,t) - T_\infty}{T_i - T_\infty} = \frac{T(x,t) - T_i}{T_\infty - T_i}$$

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

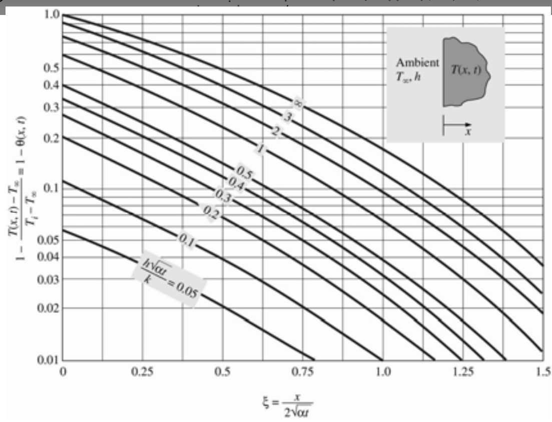


$$erfc(\xi) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-u^2} du$$

$$\frac{T(x,t) - T_i}{T_s - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

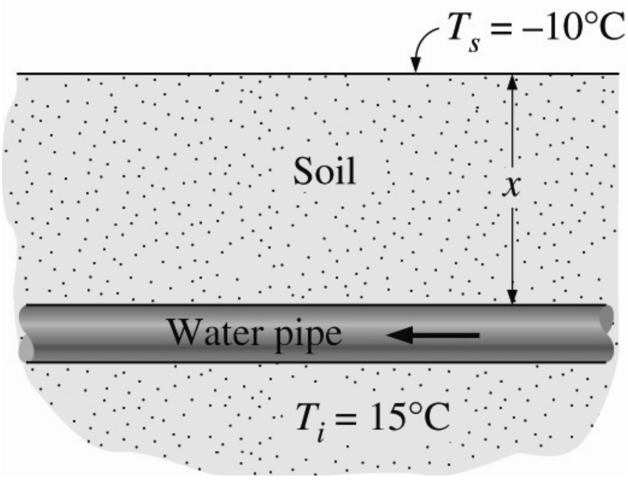
Schematic for a semi-infinite body.

FIGURE 4-23



Variation of temperature with position and time in a semi-infinite solid initially at T_i subjected to convection to an environment at T_∞ with a convection heat transfer coefficient of h (from P.J. Schneider, Ref. 10).

FIGURE 4-24



Schematic for Example 4-6.

Assumption

•The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature of –10^0C.

•The thermal properties of the soil are constant.

Analysis

$$\frac{h\sqrt{\alpha t}}{k} = \infty$$

$$1 - \frac{T(x,t) - T_\infty}{T_i - T_\infty} = 1 - \frac{0-(-10)}{15-(-10)} = 0.6$$

$$\xi = \frac{x}{2\sqrt{\alpha t}} = 0.36$$

$$t = (90 days)(24h/day)(3600 s/h) = 7.78\times 10^6 s$$

$$x = 2\xi\sqrt{\alpha t} = 2\times 0.36\sqrt{(0.15\times 10^{-6} m^2/s)(7.78\times 10^6 s)} = 0.77m$$

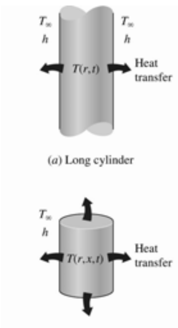
Analysis alternative solution

$$\frac{T(x,t)-T_i}{T_s - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) = 0.60$$

$$x = 2\xi\sqrt{\alpha t} = 2\times 0.37\sqrt{\left(0.15\times 10^{-6} m^2/s\right)\left(7.78\times 10^{6} s\right)} = 0.80 m$$

FIGURE 4-25

$$\left(\frac{T(r,x,t)-T_\infty}{T_i - T_\infty}\right)_{\substack{short\\cylinder}} = \left(\frac{T(x,t)-T_\infty}{T_i - T_\infty}\right)_{\substack{plane\\wall}} \left(\frac{T(r,t)-T_\infty}{T_i - T_\infty}\right)_{\substack{\inf inite\\cylinder}}$$

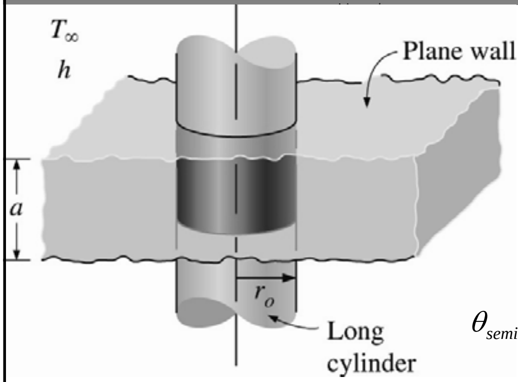


(*a*) Long cylinder

(*b*) Short cylinder (two-dimensional)

The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions.

FIGURE 4-26



$$\theta_{wall}(x,t) = \left(\frac{T(x,t)-T_\infty}{T_i - T_\infty}\right)_{plane\ wall}$$

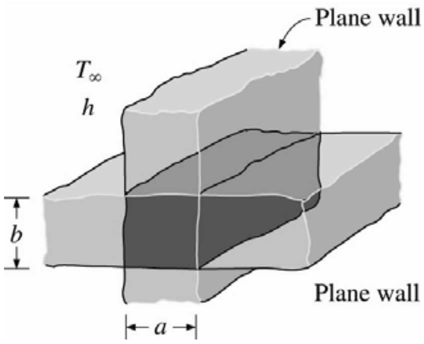
$$\theta_{cyl}(r,t) = \left(\frac{T(r,t)-T_\infty}{T_i - T_\infty}\right)_{\inf inite\ cylinder}$$

$$\theta_{semi-\inf}(x,t) = \left(\frac{T(x,t)-T_\infty}{T_i - T_\infty}\right)_{semi-\inf inite\ solid}$$

A short cylinder of radius r_0 and height *a* is the intersection of along cylinder of radius r_0 and plane wall of thickness a.

FIGURE 4-27

$$\left(\frac{T(x,y,t)-T_\infty}{T_i - T_\infty}\right)_{\text{rectangular bar}} = \theta_{wall}(x,t)\theta_{wall}(y,t)$$

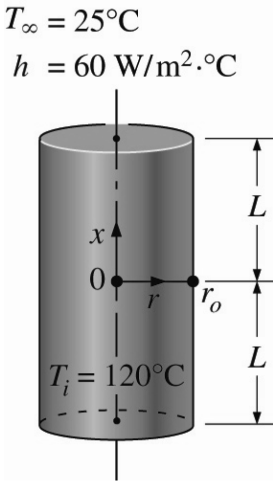


$$\left(\frac{Q}{Q_{\max}}\right)_{total,2D} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1-\left(\frac{Q}{Q_{\max}}\right)_1\right]$$

A long solid bar of rectangular profile *a* X *b* is intersection of two plane wall of thicknesses *a* and *b*.

$$\left(\frac{Q}{Q_{\max}}\right)_{total,3D} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1-\left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3\left[1-\left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1-\left(\frac{Q}{Q_{\max}}\right)_2\right]$$

FIGURE 4-28



Schematic for Example 4-7.

FIGURE 4-28

Assumption

- •Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial x- and the radial r-directions.
- • The thermal properties of the cylinder and the heat transfer coefficient are constant.
- •The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

Analysis

(a)

$$\tau = \frac{\alpha t}{L^2} = \frac{\left(3.39\times10^{-5}\, m^2/s\right)\left(900\ s\right)}{\left(0.06\ m\right)^2} = 8.48$$

$$\frac{1}{Bi} = \frac{k}{hL} = \frac{110\,W/m\bullet{}^{O}C}{\left(60\,W/m^2\cdot{}^{O}C\right)\left(0.06\,m\right)} = 30.6$$

$$\theta_{wall}(0,t) = \left(\frac{T(0,t)-T_\infty}{T_i - T_\infty}\right) = 0.8$$

$$\tau = \frac{\alpha t}{r^2} = \frac{\left(3.39\times10^{-5}\, m^2/s\right)\left(900\ s\right)}{\left(0.05\ m\right)^2} = 12.2$$

$$\frac{1}{Bi} = \frac{k}{hr_o} = \frac{110\ W\ /\ m\bullet{}^{O}C}{\left(60\ W\ /\ m^2\cdot{}^{O}C\right)\left(0.05\ m\right)} = 36.7$$

$$\theta_{cyl}(0,t) = \left(\frac{T(0,t)-T_\infty}{T_i - T_\infty}\right) = 0.5$$

51

Analysis

(a)

$$\left(\frac{T(0,0,t)-T_\infty}{T_i - T_\infty}\right)_{\substack{short\\cylinder}} = \theta_{wall}(0,t)\times\theta_{cyl}(0,t) = 0.8\times0.5 = 0.4$$

$$T(0,0,t) = T_\infty + 0.4\left(T_i - T_\infty\right)$$

$$= 25 + 0.4\left(120-25\right) = 63°C$$

52

Analysis

(b)

$$\frac{x}{L} = \frac{0.06\ m}{0.06\ m} = 1$$

$$\frac{1}{Bi} = \frac{k}{hL} = \frac{110\ W/m\bullet{}^{O}C}{\left(60\ W/m^2\cdot{}^{O}C\right)\left(0.06\ m\right)} = 30.6$$

$$\left(\frac{T(L,t) - T_\infty}{T_o - T_\infty}\right) = 0.98$$

$$\theta_{wall}(L,t) = \frac{T(L,t)-T_\infty}{T_i - T_\infty} = \left(\frac{T(L,t)-T_\infty}{T_o - T_\infty}\right)\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right) = 0.98\times 0.8 = 0.784$$

53

Analysis

(b)

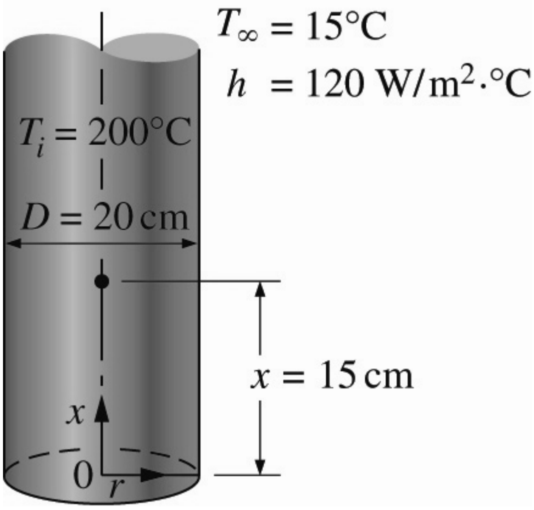
$$\left(\frac{T(L,0,t)-T_\infty}{T_i - T_\infty}\right)_{\substack{short\\cylinder}} = \theta_{wall}(L,t)\times\theta_{cyl}(0,t) = 0.784\times 0.5 = 0.392$$

$$T(L,0,t) = T_\infty + 0.392(T_i - T_\infty)$$

$$= 25 + 0.392(120 - 25) = 62.2°C$$

54

FIGURE 4-29



Schematic for Example 4-9.

FIGURE 4-28

Assumption

- Heat conduction in the semi-infinite cylinder is two dimensional, and thus the temperature varies in both the axial x- and the radial r-directions.
- The thermal properties of the cylinder and the heat transfer coefficient are constant.
- The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

Analysis

$$Bi = \frac{hr_0}{k} = \frac{(120 W/m^2 \cdot {}^{O}C)(0.1m)}{273 W/m \bullet {}^{O}C} = 0.05$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(9.71\times10^{-5} m^2/s)(5\times60\ s)}{(0.1\,m)^2} = 2.91 > 0.2$$

$$\theta_0 = \theta_{cyl}(0,t) = A_1 e^{-\lambda_1^2 \tau} = 1.0124 e^{-(0.3126)^2 (2.91)} = 0.762$$

$$1-\theta_{semi-\inf}(x,t) = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2 \alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Analysis

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{0.15\ m}{2\sqrt{(9.71\times10^{-5} m^2/s)(5\times60 s)}} = 0.44$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(120\ W/m^2 \cdot {}^{\circ}C)\sqrt{(9.71\times10^{-5} m^2/s)(300s)}}{237 W/m \cdot {}^{\circ}C} = 0.086$$

$$\frac{hx}{k} = \frac{(120\ W/m^2 \cdot {}^{\circ}C)(0.15m)}{237\ W/m^2 \cdot {}^{\circ}C} = 0.0759$$

$$\frac{h^2 \alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = (0.086)^2 = 0.0074$$

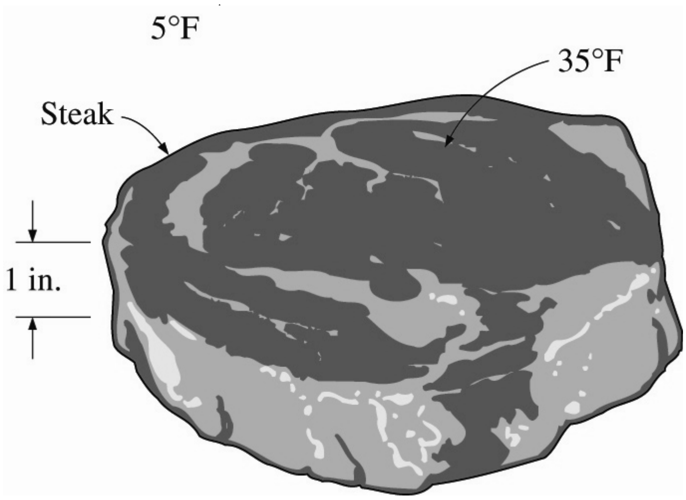
Analysis

$$\theta_{semo-\inf}(x,t) = 1 - erfc(0.44) + \exp(0.0759 + 0.0074) erfc(0.44 + 0.086)$$
$$= 1 - 0.5338 + exp(0.0833) \times 0.457$$
$$= 0.963$$

$$\left(\frac{T(x,0,t) - T_\infty}{T_i - T_\infty}\right)_{\substack{semi-\inf inte \\ cylinder}} = \theta_{semi-\inf inte}(x,t) \times \theta_{cyl}(0,t) = 0.963 \times 0.762 = 0.734$$

$$T(x,0,t) = T_\infty + 0.734(T_i - T_\infty)$$
$$= 15 + 0.734(200 - 15) = 151°C$$

FIGURE 4-30



Schematic for Example 4-10.

FIGURE 4-28

Assumption

- Heat conduction through the steaks is one-dimensional since the steaks from a large layer relative to their thickness and there is thermal symmetry about the center plane.
- The thermal properties of the steaks and the heat transfer coefficient are constant.
- The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

Analysis

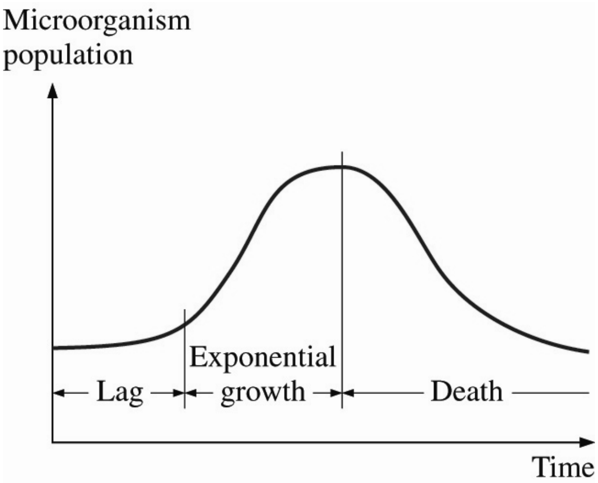
$$\frac{x}{L} = \frac{1cm}{1cm} = 1$$

$$\frac{T(L,t) - T_\infty}{T_0 - T_\infty} = \frac{2-(-15)}{7-(-15)} = 0.77$$

$$\frac{1}{Bi} = \frac{k}{hL} = 1.5$$

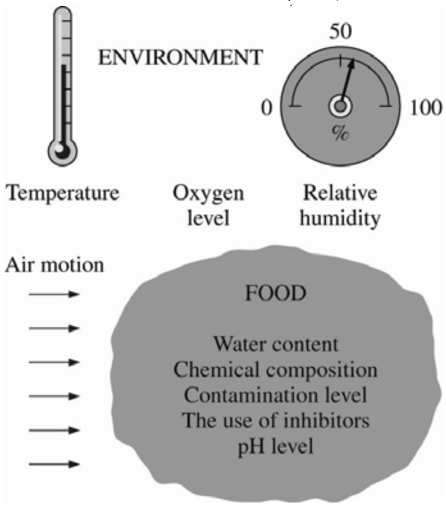
$$h = \frac{1}{1.5}\frac{k}{L} = \frac{0.45 W / cm^2 \cdot {}^O C}{1.5(1/100m)} = 30 W / cm^2 \cdot {}^O C$$

FIGURE 4-31



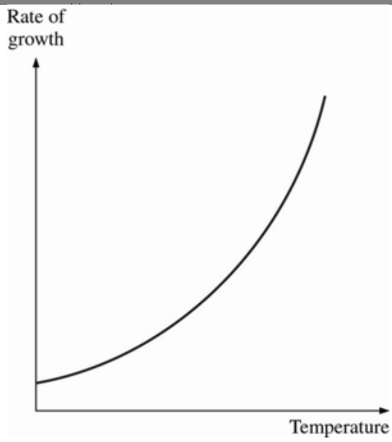
Typical growth curve of microorganisms.

FIGURE 4-32



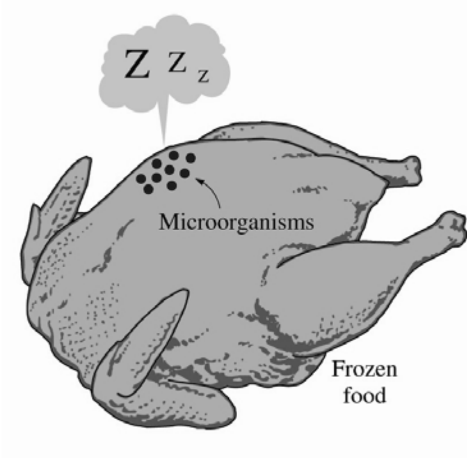
The factors that affect the rate of growth of microorganisms.

FIGURE 4-33



The rate of growth of microorganisms in a food product increases exponentially with increasing environmental temperature.

FIGURE 4-34



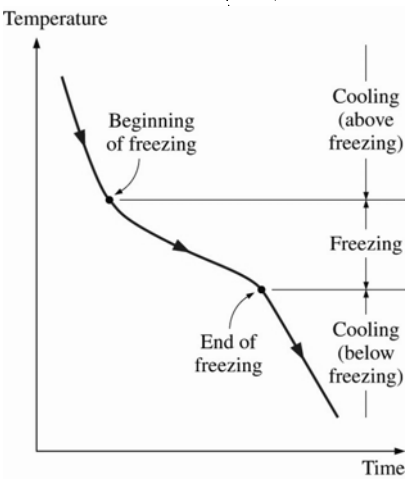
Freezing may stop the growth of microorganisms, but it may not necessarily kill them.

FIGURE 4-35

Recommended refrigeration and freezing temperatures for most perishable foods.

67

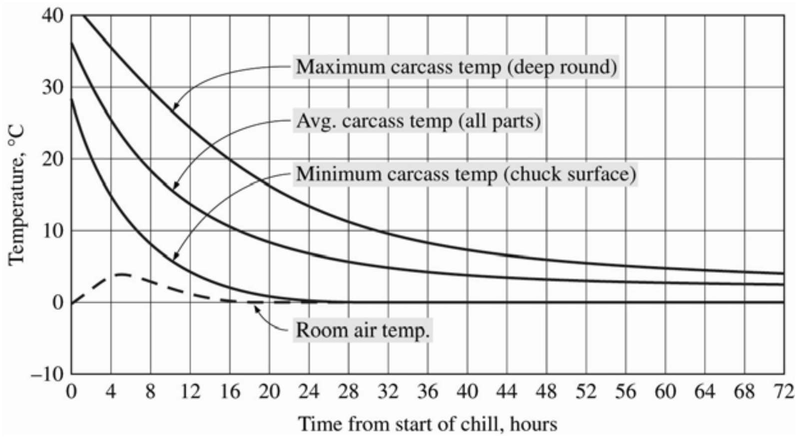
FIGURE 4-36



Typical freezing curve of food item.

68

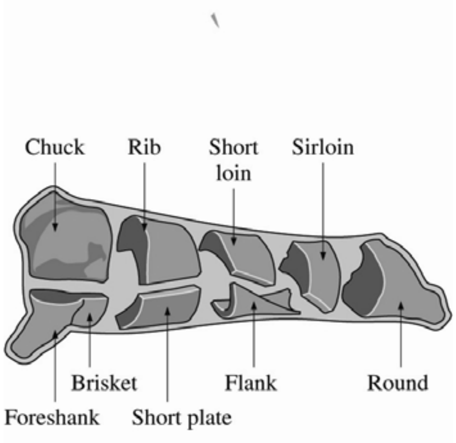
FIGURE 4-37



Typical cooling curve of a beef carcass in the chilling and holding rooms at an average temperature of 0°C (from ASHRAE, Handbook: Refrigeration, Ref. 3, Chap. 11, Fig. 2).

69

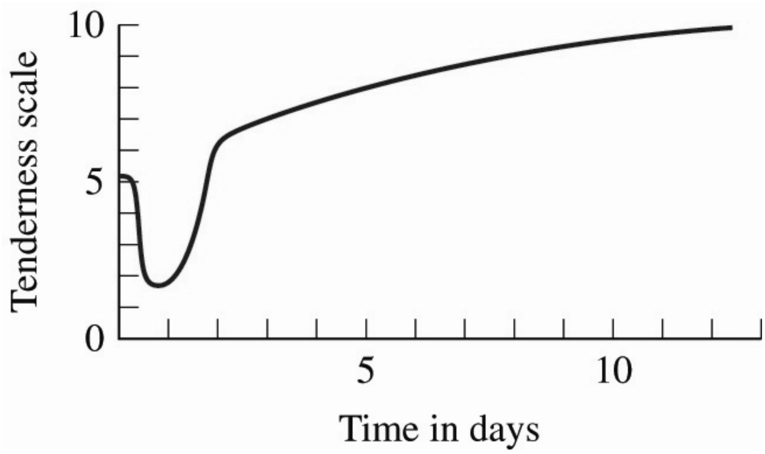
FIGURE 4-38



Various cuts of beef (from National Livestock and Meat Board).

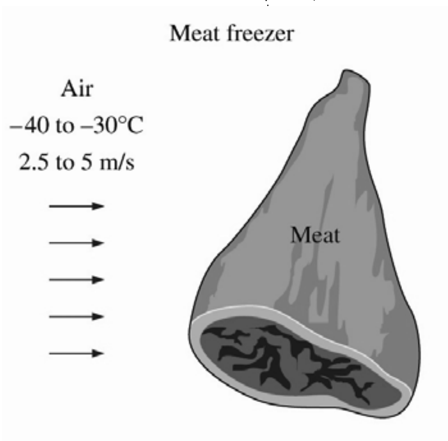
70

FIGURE 4-39



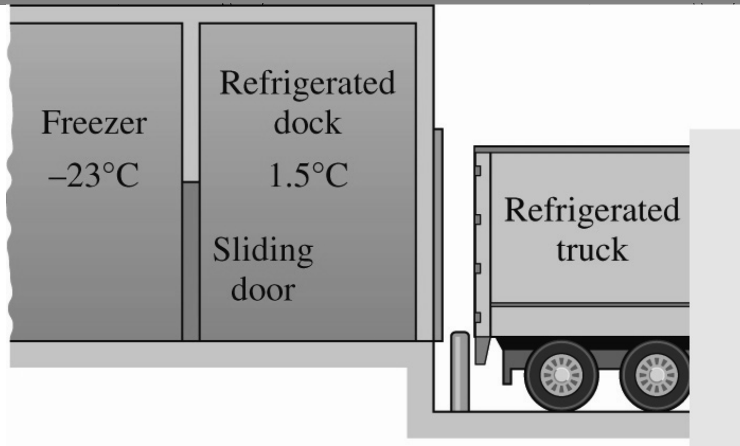
Variation of tenderness of meat stored at 2°C with time after slaughter.

FIGURE 4-40



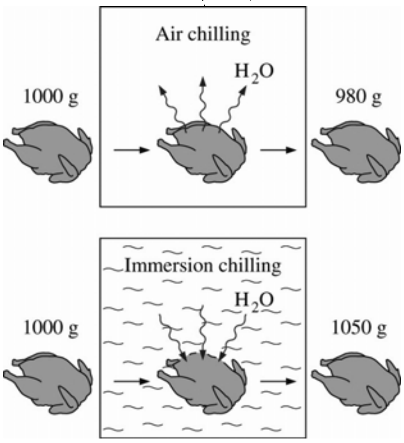
The freezing time of meat can be reduced considerably by using low temperature air at high velocity.

FIGURE 4-41



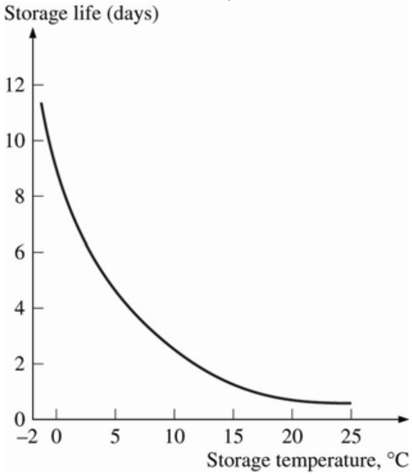
A refrigerated truck dock for loading frozen items to a refrigerated truck.

FIGURE 4-42



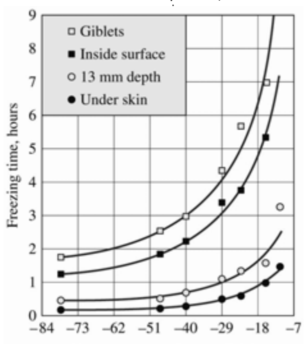
Air chilling causes dehydration and thus weight loss for poultry, whereas immersion chilling causes a weight gain as a result of water absorption.

FIGURE 4-43



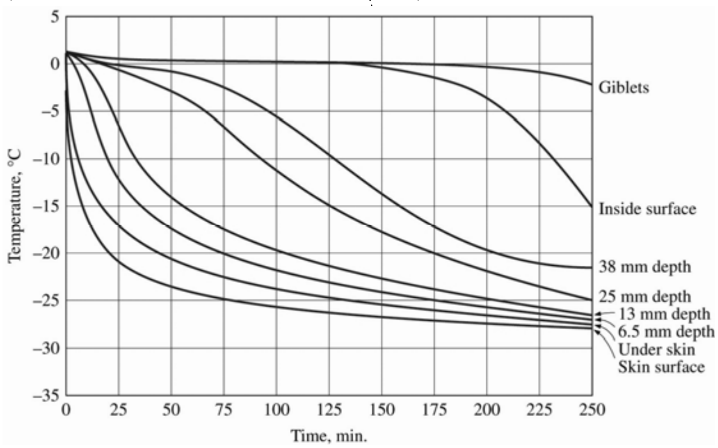
The storage life of fresh poultry decreases exponentially with increasing storage temperature.

FIGURE 4-44



The variation of freezing time of poultry with air temperature (from van der Berg and Lentz, Ref. 11).

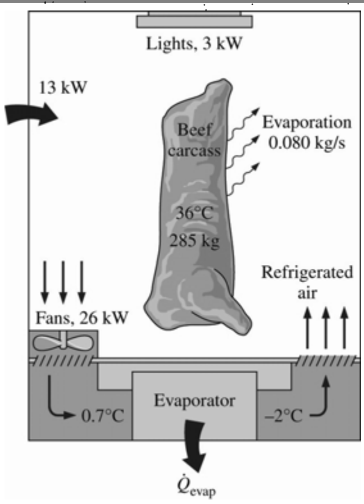
FIGURE 4-45



The variation of temperature of the breast of 6.8-kg turkeys initially at 1°C with depth during immersion cooling at -29°C (from van der Berg and Lentz, Ref. 11).

77

FIGURE 4-46



Schematic for Example 4-5.

78

FIGURE 4-28

Assumption

- Water evaporates at a rate of 0.080 kg/s.
- All the moisture in the air freezes in the evaporator.

Properties

$$C_p = 1.68 + 2.51 \times (water\ content) = 1.68 + 2.51 \times 0.58 = 3.14\ kJ/kg \cdot °C$$

Analysis (a)

$$m_{beef} = (Total\ beef\ mass\ cooled)/(Cooling\ time)$$

$$= (450\ carcasses)(285\ kg/carcass)/(10 \times 3600\ s) = 3.56\ kg/s$$

$$\dot{Q}_{beef} = (mC\Delta T)_{beef} = (3.56\ kg/s)(3.14\ kJ/kg°C)(36-15)°C = 235\ kW$$

$$\dot{Q}_{total,chillroom} = \dot{Q}_{beef} + \dot{Q}_{fan} + \dot{Q}_{lights} + \dot{Q}_{heat\ gain}$$

$$= 235 + 26 + 3 + 13 = 277\ kW$$

Analysis (b)

$$\dot{Q}_{beef,evaporative} = (mh_{fg})_{water} = (0.080\,kg/s)(2490\,kJ/kg) = 199\,kW$$

$$\dot{m}_{air} = \frac{\dot{Q}_{air}}{(C_p \Delta T_{air})} = \frac{277kW}{(1.006\,kJ/kg \cdot °C)[0.7-(-2)°C]} = 102.0\,kg/s$$

$$\Rightarrow \dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{102\,kg/s}{1.292\,kg/m^3} = 78.9m^3/s$$

81