

PROBLEM-Chapter 1-5 (Heat Transfer, Yanus A. Cengel)

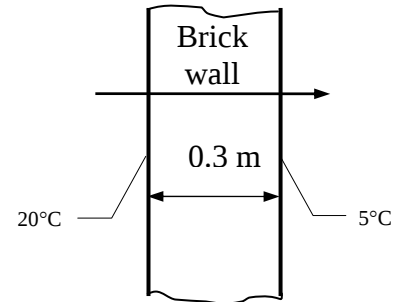
1-62 The inner and outer surfaces of a brick wall are maintained at specified temperatures. The rate of heat transfer through the wall is to be determined.

Assumptions 1 Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. 2 Thermal properties of the wall are constant.

Properties The thermal conductivity of the wall is given to be $k = 0.69 \text{ W/m}\cdot^\circ\text{C}$.

Analysis Under steady conditions, the rate of heat transfer through the wall is

$$\dot{Q}_{\text{cond}} = kA \frac{\Delta T}{L} = (0.69 \text{ W/m}\cdot^\circ\text{C})(5 \times 6 \text{ m}^2) \frac{(20 - 5)^\circ\text{C}}{0.3 \text{ m}} = \mathbf{1035 \text{ W}}$$



1-76 The heat generated in the circuitry on the surface of a 3-W silicon chip is conducted to the ceramic substrate. The temperature difference across the chip in steady operation is to be determined.

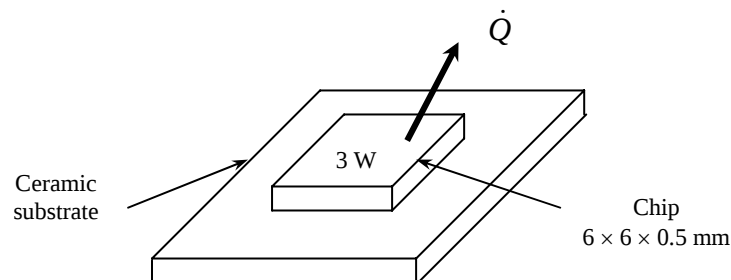
Assumptions 1 Steady operating conditions exist. 2 Thermal properties of the chip are constant.

Properties The thermal conductivity of the silicon chip is given to be $k = 130 \text{ W/m}\cdot^\circ\text{C}$.

Analysis The temperature difference between the front and back surfaces of the chip is

$$A = (0.006 \text{ m})(0.006 \text{ m}) = 0.000036 \text{ m}^2$$

$$\dot{Q} = kA \frac{\Delta T}{L} \longrightarrow \Delta T = \frac{\dot{Q}L}{kA} = \frac{(3 \text{ W})(0.0005 \text{ m})}{(130 \text{ W/m}\cdot^\circ\text{C})(0.000036 \text{ m}^2)} = \mathbf{0.32^\circ\text{C}}$$



1-102 The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis In steady operation, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{\text{generated}} = VI = (110 \text{ V})(3 \text{ A}) = 330 \text{ W}$$

The surface area of the wire is

$$A_s = (\pi D)L = \pi(0.002 \text{ m})(1.4 \text{ m}) = 0.00880 \text{ m}^2$$

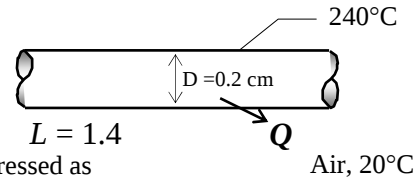
The Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Disregarding any heat transfer by radiation, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{330 \text{ W}}{(0.00880 \text{ m}^2)(240 - 20)^\circ\text{C}} = \mathbf{170.5 \text{ W/m}^2 \cdot ^\circ\text{C}}$$

Discussion If the temperature of the surrounding surfaces is equal to the air temperature in the room, the value obtained above actually represents the combined convection and radiation heat transfer coefficient.



2-23 We consider a thin spherical shell element of thickness Δr in a sphere (see Fig. 2-17 in the text). The density of the sphere is ρ , the specific heat is C , and the length is L . The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. When there is no heat generation, an *energy balance* on this thin spherical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta r(T_{t+\Delta t} - T_t)$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{Q}_{r+\Delta r} = \rho CA\Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 4\pi r^2$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \rightarrow 0$ and $\Delta t \rightarrow 0$ yields

$$\frac{1}{A} \frac{\partial}{\partial r} \left(kA \frac{\partial T}{\partial r} \right) = \rho C \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \rightarrow 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r} \left(-kA \frac{\partial T}{\partial r} \right)$$

Noting that the heat transfer area in this case is $A = 4\pi r^2$ and the thermal conductivity k is constant, the one-dimensional transient heat conduction equation in a sphere becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

where $\alpha = k / \rho C$ is the thermal diffusivity of the material.

2-24 For a medium in which the heat conduction equation is given in its simplest by $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$:

(a) Heat transfer is transient, (b) it is one-dimensional, (c) there is no heat generation, and (d) the thermal conductivity is constant.

2-25 For a medium in which the heat conduction equation is given in its simplest by

$$\frac{1}{r} \frac{d}{dr} \left(rk \frac{dT}{dr} \right) + \dot{g} = 0 :$$

(a) Heat transfer is steady, (b) it is one-dimensional, (c) there is heat generation, and (d) the thermal conductivity is variable.

2-26 For a medium in which the heat conduction equation is given by $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

(a) Heat transfer is transient, (b) it is one-dimensional, (c) there is no heat generation, and (d) the thermal conductivity is constant.

2-27 For a medium in which the heat conduction equation is given in its simplest by $r \frac{d^2 T}{dr^2} + \frac{dT}{dr} = 0 :$

(a) Heat transfer is steady, (b) it is one-dimensional, (c) there is heat generation, and (d) the thermal conductivity is constant.

2-31 For a medium in which the heat conduction equation is given by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$:

(a) Heat transfer is transient, (b) it is two-dimensional, (c) there is no heat generation, and (d) the thermal conductivity is constant.

2-77 A 2-kW resistance heater wire with a specified surface temperature is used to boil water. The center temperature of the wire is to be determined.

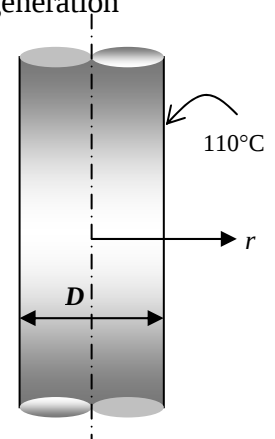
Assumptions 1 Heat transfer is steady since there is no change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. 3 Thermal conductivity is constant. 4 Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be $k = 20 \text{ W/m}\cdot^\circ\text{C}$.

Analysis The resistance heater converts electric energy into heat at a rate of 2 kW. The rate of heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{wire}} = \frac{\dot{Q}_{gen}}{\pi r_o^2 L} = \frac{2000 \text{ W}}{\pi (0.0025 \text{ m})^2 (0.7 \text{ m})} = 1.455 \times 10^8 \text{ W/m}^3$$

The center temperature of the wire is then determined from Eq. 2-71 to be



$$T_o = T_s + \frac{\dot{q} r_o^2}{4k} = 110^\circ\text{C} + \frac{(1.455 \times 10^8 \text{ W/m}^3)(0.0025 \text{ m})^2}{4(20 \text{ W/m}\cdot^\circ\text{C})} = 121.4^\circ\text{C}$$

3-1C (a) If the lateral surfaces of the rod are insulated, the heat transfer surface area of the cylindrical rod is the bottom or the top surface area of the rod, $A_s = \pi D^2 / 4$. (b) If the top and the bottom surfaces of the rod are insulated, the heat transfer area of the rod is the lateral surface area of the rod, $A = \pi DL$.

3-2C In steady heat conduction, the rate of heat transfer into the wall is equal to the rate of heat transfer out of it. Also, the temperature at any point in the wall remains constant. Therefore, the energy content of the wall does not change during steady heat conduction. However, the temperature along the wall and thus the energy content of the wall will change during transient conduction.

3-3C The temperature distribution in a plane wall will be a straight line during steady and one dimensional heat transfer with constant wall thermal conductivity.

3-4C The thermal resistance of a medium represents the resistance of that medium against heat transfer.

3-5C The combined heat transfer coefficient represents the combined effects of radiation and convection heat transfers on a surface, and is defined as $h_{\text{combined}} = h_{\text{convection}} + h_{\text{radiation}}$. It offers the convenience of incorporating the effects of radiation in the convection heat transfer coefficient, and to ignore radiation in heat transfer calculations.

3-6C Yes. The convection resistance can be defined as the inverse of the convection heat transfer coefficient per unit surface area since it is defined as $R_{\text{conv}} = 1 / (hA)$.

3-7C The convection and the radiation resistances at a surface are parallel since both the convection and radiation heat transfers occur simultaneously.

3-8C For a surface of A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad} , the single equivalent heat transfer coefficient is $h_{\text{eqv}} = h_{\text{conv}} + h_{\text{rad}}$ when the medium and the surrounding surfaces are at the same temperature. Then the equivalent thermal resistance will be $R_{\text{eqv}} = 1 / (h_{\text{eqv}} A)$.

3-9C The thermal resistance network associated with a five-layer composite wall involves five single-layer resistances connected in series.

3-22E The inner and outer surfaces of the walls of an electrically heated house remain at specified temperatures during a winter day. The amount of heat lost from the house that day and its cost are to be determined.

Assumptions 1 Heat transfer through the walls is steady since the surface temperatures of the walls remain constant at the specified values during the time period considered. 2 Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3 Thermal conductivity of the walls is constant.

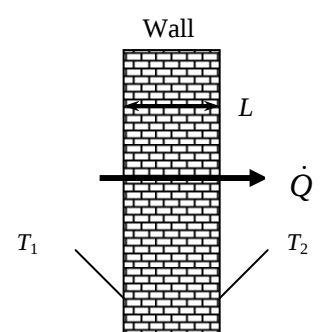
Properties The thermal conductivity of the brick wall is given to be $k = 0.40 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$.

Analysis We consider heat loss through the walls only. The total heat transfer area is

$$A = 2(40 \times 9 + 30 \times 9) = 1260 \text{ ft}^2$$

The rate of heat loss during the daytime is

$$\dot{Q}_{\text{day}} = kA \frac{T_1 - T_2}{L} = (0.40 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(1260 \text{ ft}^2) \frac{(55 - 45)^\circ\text{F}}{1 \text{ ft}} = 5040 \text{ Btu/h}$$



The rate of heat loss during nighttime is

$$\begin{aligned}\dot{Q}_{\text{night}} &= kA \frac{T_1 - T_2}{L} \\ &= (0.40 \text{ Btu/h.ft.}^\circ\text{F})(1260 \text{ ft}^2) \frac{(55 - 35)^\circ\text{C}}{1 \text{ ft}} = 10,080 \text{ Btu/h}\end{aligned}$$

The amount of heat loss from the house that night will be

$$\begin{aligned}\dot{Q} &= \frac{Q}{\Delta t} \longrightarrow Q = \dot{Q} \Delta t = 10\dot{Q}_{\text{day}} + 14\dot{Q}_{\text{night}} = (10 \text{ h})(5040 \text{ Btu/h}) + (14 \text{ h})(10,080 \text{ Btu/h}) \\ &= \mathbf{191,520 \text{ Btu}}\end{aligned}$$

Then the cost of this heat loss for that day becomes

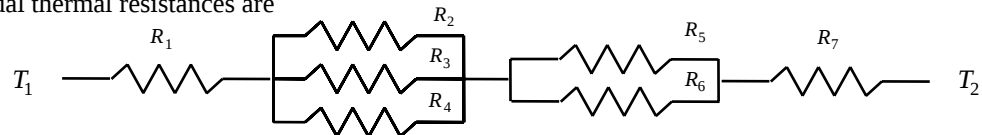
$$\text{Cost} = (191,520 / 3412 \text{ kWh})(\$0.09 / \text{kWh}) = \mathbf{\$5.05}$$

3-57 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of change with time. 2 Heat transfer through the wall is one-dimensional. 3 Thermal conductivities are constant. 4 Thermal contact resistances at the interfaces are disregarded.

Properties The thermal conductivities are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35 \text{ W/m}^\circ\text{C}$.

Analysis (a) The representative surface area is $A = 0.12 \times 1 = 0.12 \text{ m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA} \right)_A = \frac{0.01 \text{ m}}{(2 \text{ W/m}^\circ\text{C})(0.12 \text{ m}^2)} = 0.04^\circ\text{C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA} \right)_C = \frac{0.05 \text{ m}}{(20 \text{ W/m}^\circ\text{C})(0.04 \text{ m}^2)} = 0.06^\circ\text{C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA} \right)_B = \frac{0.05 \text{ m}}{(8 \text{ W/m}^\circ\text{C})(0.04 \text{ m}^2)} = 0.16^\circ\text{C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA} \right)_D = \frac{0.1 \text{ m}}{(15 \text{ W/m}^\circ\text{C})(0.06 \text{ m}^2)} = 0.11^\circ\text{C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA} \right)_E = \frac{0.1 \text{ m}}{(35 \text{ W/m}^\circ\text{C})(0.06 \text{ m}^2)} = 0.05^\circ\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA} \right)_F = \frac{0.06 \text{ m}}{(2 \text{ W/m}^\circ\text{C})(0.12 \text{ m}^2)} = 0.25^\circ\text{C/W}$$

$$\frac{1}{R_{\text{mid},1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{\text{mid},1} = 0.025^\circ\text{C/W}$$

$$\frac{1}{R_{\text{mid},2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{\text{mid},2} = 0.034^\circ\text{C/W}$$

$$R_{\text{total}} = R_1 + R_{\text{mid},1} + R_{\text{mid},2} + R_7 = 0.04 + 0.025 + 0.034 + 0.25 = 0.349^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(300 - 100)^\circ\text{C}}{0.349^\circ\text{C/W}} = 572 \text{ W (for a } 0.12 \text{ m} \times 1 \text{ m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{\text{total}} = (572 \text{ W}) \frac{(5 \text{ m})(8 \text{ m})}{0.12 \text{ m}^2} = \mathbf{1.91 \times 10^5 \text{ W}}$$

(b) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065 \text{ } ^\circ\text{C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300^\circ\text{C} - (572 \text{ W})(0.065 \text{ } ^\circ\text{C/W}) = \mathbf{263^\circ\text{C}}$$

(c) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (572 \text{ W})(0.25 \text{ } ^\circ\text{C/W}) = \mathbf{143^\circ\text{C}}$$

3-75E A steam pipe covered with 2-in thick fiberglass insulation is subjected to convection on its surfaces. The rate of heat loss from the steam per unit length and the error involved in neglecting the thermal resistance of the steel pipe in calculations are to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. 3 Thermal conductivities are constant. 4 The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 8.7 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ for steel and $k = 0.020 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ for fiberglass insulation.

Analysis The inner and outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(3.5/12 \text{ ft})(1 \text{ ft}) = 0.916 \text{ ft}^2$$

$$A_o = \pi D_o L = \pi(8/12 \text{ ft})(1 \text{ ft}) = 2.094 \text{ ft}^2$$

$$T_{\infty 1} \begin{array}{c} R_i \\ \text{---} \end{array} \begin{array}{c} R_{\text{pipe}} \\ \text{---} \end{array} \begin{array}{c} R_{\text{insulation}} \\ \text{---} \end{array} \begin{array}{c} R_o \\ \text{---} \end{array} T_{\infty 2}$$

The individual resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(30 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(0.916 \text{ ft}^2)} = 0.036 \text{ h}\cdot^\circ\text{F/Btu}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2 / r_1)}{2\pi k_1 L} = \frac{\ln(2 / 1.75)}{2\pi(8.7 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(1 \text{ ft})} = 0.002 \text{ h}\cdot^\circ\text{F/Btu}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3 / r_2)}{2\pi k_2 L} = \frac{\ln(4 / 2)}{2\pi(0.020 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(1 \text{ ft})} = 5.516 \text{ h}\cdot^\circ\text{F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(5 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(2.094 \text{ ft}^2)} = 0.096 \text{ h}\cdot^\circ\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.036 + 0.002 + 5.516 + 0.096 = 5.65 \text{ h}\cdot^\circ\text{F/Btu}$$

Then the steady rate of heat loss from the steam per ft. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(450 - 55)^\circ\text{F}}{5.65 \text{ h}\cdot^\circ\text{F/Btu}} = \mathbf{69.91 \text{ Btu/h}}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_i + R_2 + R_o = 0.036 + 5.516 + 0.096 = 5.648 \text{ h}\cdot^\circ\text{F/Btu}$$

Then the percentage error involved in calculations becomes

$$\text{error}\% = \frac{(5.65 - 5.648) \text{ h}\cdot^\circ\text{F/Btu}}{5.65 \text{ h}\cdot^\circ\text{F/Btu}} \times 100 = \mathbf{0.035\%}$$

which is insignificant.

4-14 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

Assumptions **1** The junction is spherical in shape with a diameter of $D = 0.0012$ m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35$ W/m $\cdot$ °C, $\rho = 8500$ kg/m 3 , and $C_p = 320$ J/kg $\cdot$ °C.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{0.0012 \text{ m}}{6} = 0.0002 \text{ m}$$

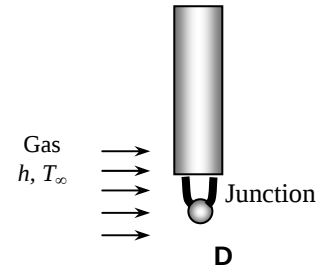
$$Bi = \frac{hL_c}{k} = \frac{(65 \text{ W/m}^2 \cdot \text{°C})(0.0002 \text{ m})}{(35 \text{ W/m} \cdot \text{°C})} = 0.00037 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{65 \text{ W/m}^2 \cdot \text{°C}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg} \cdot \text{°C})(0.0002 \text{ m})} = 0.1195 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1195 \text{ s}^{-1})t} \longrightarrow t = \mathbf{38.5 \text{ s}}$$



4-34 An egg is dropped into boiling water. The cooking time of the egg is to be determined. ✓

Assumptions 1 The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. 2 Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. 3 The thermal properties of the egg are constant. 4 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be $k = 0.6$ W/m.°C and $\alpha = 0.14 \times 10^{-6}$ m²/s.

Analysis The Biot number for this process is

$$Bi = \frac{hr_0}{k} = \frac{(1400 \text{ W/m}^2 \cdot ^\circ\text{C})(0.0275 \text{ m})}{(0.6 \text{ W/m} \cdot ^\circ\text{C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

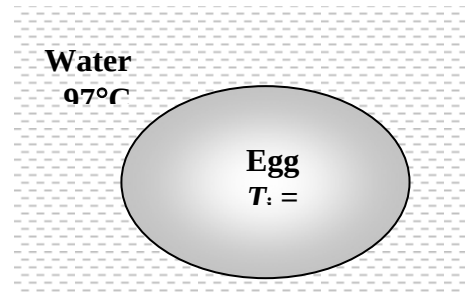
$$\lambda_1 = 3.0877 \quad \text{and} \quad A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 97}{8 - 97} = (1.9969) e^{-(3.0877)^2 \tau} \longrightarrow \tau = 0.198 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

$$t = \frac{\tau r_0^2}{\alpha} = \frac{(0.198)(0.0275 \text{ m})^2}{(0.14 \times 10^{-6} \text{ m}^2/\text{s})} = 1068 \text{ s} = \mathbf{17.8 \text{ min}}$$



4-54 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions 1 The potatoes are spherical in shape with a radius of $r_0 = 3$ cm. 2 Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. 3 The thermal properties of the potato are constant. 4 The heat transfer coefficient is constant and uniform over the entire surface. 5 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be $k = 0.50$ W/m.°C and $\alpha = 0.13 \times 10^{-6}$ m²/s.

Analysis First we find the Biot number:

$$Bi = \frac{hr_0}{k} = \frac{(19 \text{ W/m}^2 \cdot ^\circ\text{C})(0.03 \text{ m})}{0.5 \text{ W/m} \cdot ^\circ\text{C}} = 1.14$$

From Table 4-1 we read, for a sphere, $\lambda_1 = 1.635$ and $A_1 = 1.302$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{6 - 2}{25 - 2} = 1.302 e^{-(1.635)^2 \tau} \longrightarrow \tau = 0.753$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

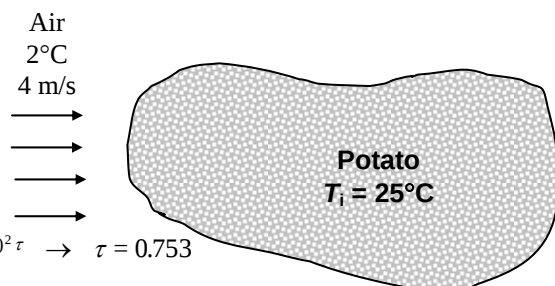
$$\tau = \frac{\alpha t}{r_0^2} \longrightarrow t = \frac{\tau r_0^2}{\alpha} = \frac{(0.753)(0.03 \text{ m})^2}{0.13 \times 10^{-6} \text{ m}^2/\text{s}} = 5213 \text{ s} = \mathbf{1.45 \text{ h}}$$

The lowest temperature during cooling will occur on the surface ($r/r_0 = 1$), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r / r_0)}{\lambda_1 r / r_0} \longrightarrow \frac{T(r_0) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_0 / r_0)}{\lambda_1 r_0 / r_0} = \frac{T_0 - T_\infty}{T_i - T_\infty} \frac{\sin(\lambda_1 r_0 / r_0)}{\lambda_1 r_0 / r_0}$$

$$\frac{T(r_0) - 2}{25 - 2} = \left(\frac{6 - 2}{25 - 2} \right) \frac{\sin(1.635 \text{ rad})}{1.635} \longrightarrow T(r_0) = 4.44^\circ\text{C}$$

Substituting,



which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, no part of the potatoes will experience chilling injury during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left. \begin{aligned} \frac{1}{Bi} = \frac{k}{hr_o} &= \frac{0.50 \text{ W/m} \cdot ^\circ\text{C}}{(19 \text{ W/m}^2 \cdot ^\circ\text{C})(0.03 \text{ m})} = 0.877 \\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{6 - 2}{25 - 2} = 0.174 \end{aligned} \right\} \tau = \frac{\alpha t}{r_o^2} = 0.75 \quad (\text{Fig. 4-15a})$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.75)(0.03)^2}{0.13 \times 10^{-6} \text{ m}^2/\text{s}} = 5192 \text{ s} \approx \mathbf{1.44 \text{ h}}$$

Therefore,

The surface temperature is determined from

$$\left. \begin{aligned} \frac{1}{Bi} = \frac{k}{hr_o} &= 0.877 \\ \frac{r}{r_o} &= 1 \end{aligned} \right\} \frac{T(r) - T_\infty}{T_o - T_\infty} = 0.6 \quad (\text{Fig. 4-15b})$$

which gives $T_{\text{surface}} = T_\infty + 0.6(T_o - T_\infty) = 2 + 0.6(6 - 2) = 4.4^\circ\text{C}$

The slight difference between the two results is due to the reading error of the charts.

4-47 A hot dog is dropped into boiling water, and temperature measurements are taken at certain time intervals. The thermal diffusivity and thermal conductivity of the hot dog and the convection heat transfer coefficient are to be determined.

Assumptions 1 Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. 2 The thermal properties of the hot dog are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

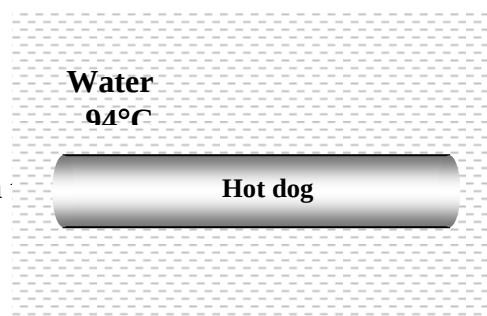
Properties The properties of hot dog available are given to be $\rho = 980 \text{ kg/m}^3$ and $C_p = 3900 \text{ J/kg} \cdot ^\circ\text{C}$.

Analysis (a) From Fig. 4-14b we have

$$\left. \begin{aligned} \frac{T - T_\infty}{T_o - T_\infty} &= \frac{88 - 94}{59 - 94} = 0.17 \\ \frac{r}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned} \right\} \frac{1}{Bi} = \frac{k}{hr_o} = 0.15$$

The Fourier number is determined from Fig. 4-14a

$$\left. \begin{aligned} \frac{1}{Bi} = \frac{k}{hr_o} &= 0.15 \\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{59 - 94}{20 - 94} = 0.47 \end{aligned} \right\} \tau = \frac{\alpha t}{r_o^2} = 0.20$$



The thermal diffusivity of the hot dog is determined to be

$$\frac{\alpha t}{r_o^2} = 0.20 \longrightarrow \alpha = \frac{0.2 r_o^2}{t} = \frac{(0.2)(0.011 \text{ m})^2}{120 \text{ s}} = \mathbf{2.017 \times 10^{-7} \text{ m}^2/\text{s}}$$

(b) The thermal conductivity of the hot dog is determined from

$$k = \alpha \rho C_p = (2.017 \times 10^{-7} \text{ m}^2/\text{s})(980 \text{ kg/m}^3)(3900 \text{ J/kg} \cdot ^\circ\text{C}) = \mathbf{0.771 \text{ W/m} \cdot ^\circ\text{C}}$$

(c) From part (a) we have $\frac{1}{Bi} = \frac{k}{hr_o} = 0.15$. Then,

$$\frac{k}{h} = 0.15r_o = (0.15)(0.011\text{ m}) = 0.00165\text{ m}$$

Therefore, the heat transfer coefficient is

$$\frac{k}{h} = 0.00165 \longrightarrow h = \frac{0.771\text{ W/m}\cdot^\circ\text{C}}{0.00165\text{ m}} = \mathbf{467\text{ W/m}^2\cdot^\circ\text{C}}$$

4-50 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions 1 The apples are spherical in shape with a diameter of 9 cm. 2 Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. 3 The thermal properties of the apples are constant. 4 The heat transfer coefficient is constant and uniform over the entire surface. 5 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418\text{ W/m}\cdot^\circ\text{C}$, $\rho = 840\text{ kg/m}^3$, $C_p = 3.81\text{ kJ/kg}\cdot^\circ\text{C}$, and $\alpha = 1.3 \times 10^{-7}\text{ m}^2/\text{s}$.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\text{ W/m}^2\cdot^\circ\text{C})(0.045\text{ m})}{(0.418\text{ W/m}\cdot^\circ\text{C})} = 0.861$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3 \times 10^{-7}\text{ m}^2/\text{s})(1\text{ h} \times 3600\text{ s/h})}{(0.045\text{ m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{o,sph} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_o - (-15)}{20 - (-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_o = \mathbf{11.2^\circ\text{C}}$$

The temperature at the surface of the apples is

$$\theta(r_o, t)_{sph} = \frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)} \frac{\sin(1.476\text{ rad})}{1.476} = 0.505$$

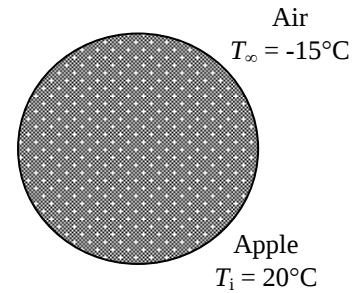
$$\frac{T(r_o, t) - (-15)}{20 - (-15)} = 0.505 \longrightarrow T(r_o, t) = \mathbf{2.7^\circ\text{C}}$$

The maximum possible heat transfer is

$$m = \rho V = \rho \frac{4}{3} \pi r_o^3 = (840\text{ kg/m}^3) \left[\frac{4}{3} \pi (0.045\text{ m})^3 \right] = 0.3206\text{ kg}$$

$$Q_{\max} = m C_p (T_i - T_\infty) = (0.3206\text{ kg})(3.81\text{ kJ/kg}\cdot^\circ\text{C})[20 - (-15)]^\circ\text{C} = 42.76\text{ kJ}$$

Then the actual amount of heat transfer becomes



$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o, sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749) \frac{\sin(1.476 \text{ rad}) - (1.476) \cos(1.476 \text{ rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402 Q_{\max} = (0.402)(42.76 \text{ kJ}) = \mathbf{17.2 \text{ kJ}}$$

4-67 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

Assumptions 1 The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. 2 The thermal properties of the wood slab are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are $k = 0.17 \text{ W/m} \cdot ^\circ\text{C}$ and $\alpha = 1.28 \times 10^{-7} \text{ m}^2/\text{s}$.

Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right) \right]$$

where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(35 \text{ W/m}^2 \cdot ^\circ\text{C})\sqrt{(1.28 \times 10^{-7} \text{ m}^2/\text{s})(5 \times 60 \text{ s})}}{0.17 \text{ W/m} \cdot ^\circ\text{C}} = 1.276$$

$$\frac{h^2 \alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 1.276^2 = 1.628$$

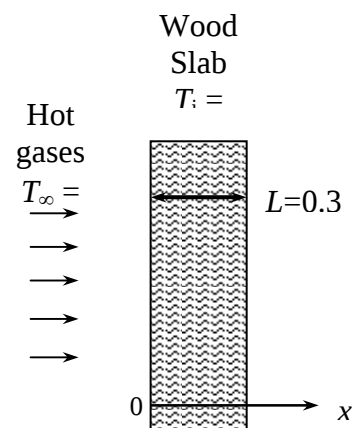
Noting that $x = 0$ at the surface and using Table 4-3 for *erfc* values,

$$\begin{aligned} \frac{T(x, t) - 25}{550 - 25} &= \text{erfc}(0) - \exp(0 + 1.628) \text{erfc}(0 + 1.276) \\ &= 1 - (5.0937)(0.0727) \\ &= 0.630 \end{aligned}$$

Solving for $T(x, t)$ gives

$$T(x, t) = \mathbf{356^\circ\text{C}}$$

which is less than the ignition temperature of 450°C . Therefore, the wood will not ignite.



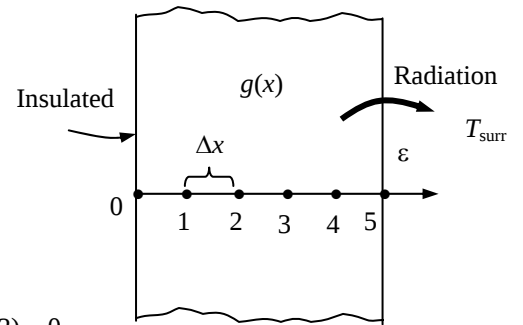
5-18 A plane wall with variable heat generation and constant thermal conductivity is subjected to insulation at the left (node 0) and radiation at the right boundary (node 5). The finite difference formulation of the boundary nodes is to be determined.

Assumptions 1 Heat transfer through the wall is given to be steady and one-dimensional, and the thermal conductivity to be constant. 2 Convection heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become

Left boundary node:
$$kA \frac{T_1 - T_0}{\Delta x} + \dot{q}_0 (A\Delta x / 2) = 0$$

Right boundary node:
$$\varepsilon \sigma A (T_{\text{surr}}^4 - T_5^4) + kA \frac{T_4 - T_5}{\Delta x} + \dot{q}_5 (A\Delta x / 2) = 0$$



5-24 A uranium plate is subjected to insulation on one side and convection on the other. The finite difference formulation of this problem is to be obtained, and the nodal temperatures under steady conditions are to be determined.

Assumptions 1 Heat transfer through the wall is steady since there is no indication of change with time. 2 Heat transfer is one-dimensional since the plate is large relative to its thickness. 3 Thermal conductivity is constant. 4 Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be $k = 28 \text{ W/m} \cdot ^\circ\text{C}$.

Analysis The number of nodes is specified to be $M = 6$. Then the nodal spacing Δx becomes

$$\Delta x = \frac{L}{M-1} = \frac{0.05 \text{ m}}{6-1} = 0.01 \text{ m}$$

This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Node 0 is on insulated boundary, and thus we can treat it as an interior node by using the mirror image concept. Nodes 1, 2, 3, and 4 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0, \quad \text{for } m = 0, 1, 2, 3, \text{ and } 4$$

Finally, the finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 5 and taking the direction of all heat transfers to be towards the node under consideration:

Node 0 (Left surface - insulated): $\frac{T_1 - 2T_0 + T_1}{\Delta x^2} + \frac{\dot{g}}{k} = 0$

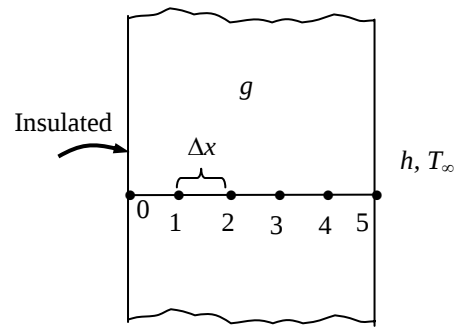
Node 1 (interior): $\frac{T_0 - 2T_1 + T_2}{\Delta x^2} + \frac{\dot{g}}{k} = 0$

Node 2 (interior): $\frac{T_1 - 2T_2 + T_3}{\Delta x^2} + \frac{\dot{g}}{k} = 0$

Node 3 (interior): $\frac{T_2 - 2T_3 + T_4}{\Delta x^2} + \frac{\dot{g}}{k} = 0$

Node 4 (interior): $\frac{T_3 - 2T_4 + T_5}{\Delta x^2} + \frac{\dot{g}}{k} = 0$

Node 5 (right surface - convection): $h(T_\infty - T_5) + k \frac{T_4 - T_5}{\Delta x} + \dot{g}(\Delta x / 2) = 0$



where $\Delta x = 0.01 \text{ m}$, $\dot{g} = 6 \times 10^5 \text{ W/m}^3$, $k = 28 \text{ W/m} \cdot ^\circ\text{C}$, $h = 60 \text{ W/m}^2 \cdot ^\circ\text{C}$, and $T_\infty = 30^\circ\text{C}$. This system of 6 equations with six unknown temperatures constitute the finite difference formulation of the problem.

(b) The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

$$T_0 = 556.8^\circ\text{C}, \quad T_1 = 555.7^\circ\text{C}, \quad T_2 = 552.5^\circ\text{C}, \quad T_3 = 547.1^\circ\text{C}, \quad T_4 = 539.6^\circ\text{C}, \quad \text{and} \quad T_5 = 530.0^\circ\text{C}$$

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-29 A plate is subjected to specified heat flux and specified temperature on one side, and no conditions on the other. The finite difference formulation of this problem is to be obtained, and the temperature of the other side under steady conditions is to be determined.

Assumptions 1 Heat transfer through the plate is given to be steady and one-dimensional. 2 There is no heat generation in the plate.

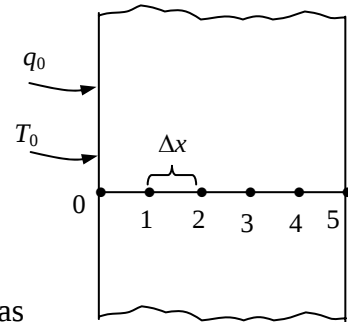
Properties The thermal conductivity is given to be $k = 2.5 \text{ W/m}\cdot^\circ\text{C}$.

Analysis The nodal spacing is given to be $\Delta x = 0.06 \text{ m}$.

Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{0.3 \text{ m}}{0.06 \text{ m}} + 1 = 6$$

Nodes 1, 2, 3, and 4 are interior nodes, and thus for them



we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0 \rightarrow T_{m+1} - 2T_m + T_{m-1} = 0 \quad (\text{since } \dot{g} = 0), \quad \text{for } m = 1, 2, 3, \text{ and } 4$$

The finite difference equation for node 0 on the left surface is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration,

$$\dot{q}_0 + k \frac{T_1 - T_0}{\Delta x} = 0 \rightarrow 700 \text{ W/m}^2 + (2.5 \text{ W/m}\cdot^\circ\text{C}) \frac{T_1 - 60^\circ\text{C}}{0.06 \text{ m}} = 0 \rightarrow T_1 = 43.2^\circ\text{C}$$

Other nodal temperatures are determined from the general interior node relation as follows:

$$\begin{aligned} m = 1: & \quad T_2 = 2T_1 - T_0 = 2 \times 43.2 - 60 = 26.4^\circ\text{C} \\ m = 2: & \quad T_3 = 2T_2 - T_1 = 2 \times 26.4 - 43.2 = 9.6^\circ\text{C} \\ m = 3: & \quad T_4 = 2T_3 - T_2 = 2 \times 9.6 - 26.4 = -7.2^\circ\text{C} \\ m = 4: & \quad T_5 = 2T_4 - T_3 = 2 \times (-7.2) - 9.6 = -24^\circ\text{C} \end{aligned}$$

Therefore, the temperature of the other surface will be -24°C

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-84 A uranium plate initially at a uniform temperature is subjected to insulation on one side and convection on the other. The transient finite difference formulation of this problem is to be obtained, and the nodal temperatures after 5 min and under steady conditions are to be determined.

Assumptions 1 Heat transfer is one-dimensional since the plate is large relative to its thickness. 2 Thermal conductivity is constant. 3 Radiation heat transfer is negligible.

Properties The conductivity and diffusivity are given to be $k = 28 \text{ W/m}\cdot^\circ\text{C}$ and $\alpha = 12.5 \times 10^{-6} \text{ m}^2/\text{s}$.

Analysis The nodal spacing is given to be $\Delta x = 0.02 \text{ m}$. Then the number of nodes becomes $M = L/\Delta x + 1 = 0.08/0.02 + 1 = 5$. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. Node 0 is on insulated boundary, and thus we can treat it as an interior node by using the mirror image concept. Nodes

1, 2, and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\rightarrow T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1 - 2\tau)T_m^i + \tau \frac{\dot{g}_m^i \Delta x^2}{k}$$

The finite difference equation for node 4 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 4 and taking the direction of all heat transfers to be towards the node under

consideration:

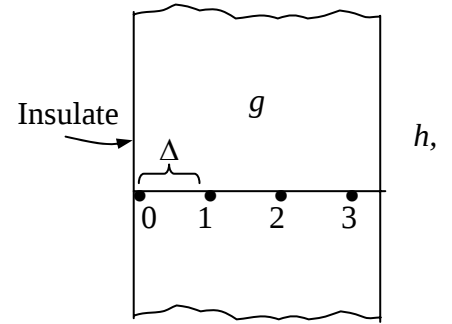
Node 0 (insulated): $T_0^{i+1} = \tau(T_1^i + T_1^i) + (1 - 2\tau)T_0^i + \tau \frac{\dot{g}_0 \Delta x^2}{k}$

Node 1 (interior): $T_1^{i+1} = \tau(T_0^i + T_2^i) + (1 - 2\tau)T_1^i + \tau \frac{\dot{g}_0 \Delta x^2}{k}$

Node 2 (interior): $T_2^{i+1} = \tau(T_1^i + T_3^i) + (1 - 2\tau)T_2^i + \tau \frac{\dot{g}_0 \Delta x^2}{k}$

Node 3 (interior): $T_3^{i+1} = \tau(T_2^i + T_4^i) + (1 - 2\tau)T_3^i + \tau \frac{\dot{g}_0 \Delta x^2}{k}$

Node 4 (convection): $h(T_\infty - T_4^i) + k \frac{T_3^i - T_4^i}{\Delta x} + \dot{g}_0 \frac{\Delta x}{2} = \rho \frac{\Delta x}{2} C \frac{T_4^{i+1} - T_4^i}{\Delta t}$



or

$$T_4^{i+1} = \left(1 - 2\tau - 2\tau \frac{h\Delta x}{k}\right)T_4^i + 2\tau T_3^i + 2\tau \frac{h\Delta x}{k}T_\infty + \tau \frac{\dot{g}_0 (\Delta x)^2}{k}$$

where $\Delta x = 0.02 \text{ m}$, $\dot{g}_0 = 10^6 \text{ W/m}^3$, $k = 28 \text{ W/m} \cdot ^\circ\text{C}$, $h = 35 \text{ W/m}^2 \cdot ^\circ\text{C}$, $T_\infty = 20^\circ\text{C}$, and $\alpha = 12.5 \times 10^{-6} \text{ m}^2/\text{s}$. The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_4^i is smaller in this case, and thus the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau \frac{h\Delta x}{k} \geq 0 \rightarrow \tau \leq \frac{1}{2(1 + h\Delta x / k)} \rightarrow \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x / k)}$$

since $\tau = \alpha \Delta t / \Delta x^2$. Substituting the given quantities, the maximum allowable the time step becomes

$$\Delta t \leq \frac{(0.02 \text{ m})^2}{2(12.5 \times 10^{-6} \text{ m}^2/\text{s})[1 + (35 \text{ W/m}^2 \cdot ^\circ\text{C})(0.02 \text{ m})/(28 \text{ W/m} \cdot ^\circ\text{C})]} = 15.6 \text{ s}$$

Therefore, any time step less than 15.5 s can be used to solve this problem. For convenience, let us choose the time step to be $\Delta t = 15 \text{ s}$. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(12.5 \times 10^{-6} \text{ m}^2/\text{s})(15 \text{ s})}{(0.02 \text{ m})^2} = 0.46875$$

Substituting this value of τ and other given quantities, the nodal temperatures after $5 \times 60 / 15 = 20$ time steps (5 min) are determined to be

After 5 min: $T_0 = 228.9^\circ\text{C}$, $T_1 = 228.4^\circ\text{C}$, $T_2 = 226.8^\circ\text{C}$, $T_3 = 224.0^\circ\text{C}$, and $T_4 = 219.9^\circ\text{C}$

(b) The time needed for transient operation to be established is determined by increasing the number of time steps until the nodal temperatures no longer change. In this case steady operation is established in ---- min, and the nodal temperatures under steady conditions are determined to be

$T_0 = 2420^\circ\text{C}$, $T_1 = 2413^\circ\text{C}$, $T_2 = 2391^\circ\text{C}$, $T_3 = 2356^\circ\text{C}$, and $T_4 = 2306^\circ\text{C}$

Discussion The steady solution can be checked independently by obtaining the steady finite difference formulation, and solving the resulting equations simultaneously.

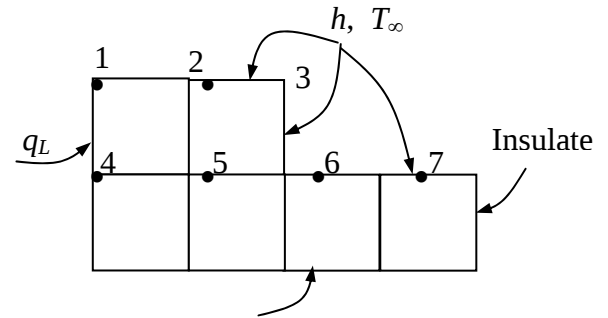
5-87 Heat conduction through a long L-shaped solid bar with specified boundary conditions is considered. The temperature at the top corner (node #3) of the body after 2, 5, and 30 min is to be determined with the transient explicit finite difference method.

Assumptions 1 Heat transfer through the body is given to be transient and two-dimensional. 2 Thermal conductivity is constant. 3 Heat generation is uniform.

Properties The conductivity and diffusivity are given to be $k = 15 \text{ W/m}\cdot^\circ\text{C}$ and $\alpha = 3.2 \times 10^{-6} \text{ m}^2/\text{s}$.

Analysis The nodal spacing is given to be $\Delta x = \Delta y = l = 0.015 \text{ m}$. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{G}_{\text{element}}^i = \rho V_{\text{element}} C \frac{T_m^{i+1} - T_m^i}{\Delta t}$$



The quantities h , T_∞ , $\dot{g}$, and $\dot{q}_R$ do not change with time, and thus we do not need to use the superscript i for them. Also, the energy balance expressions can be simplified using the definitions of thermal diffusivity $\alpha = k / (\rho C)$ and the dimensionless mesh Fourier number $\tau = \alpha \Delta t / l^2$ where $\Delta x = \Delta y = l$. We note that all nodes are boundary nodes except node 5 that is an interior node. Therefore, we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 8 nodes are obtained as follows:

$$\text{Node 1: } \dot{q}_L \frac{l}{2} + h \frac{l}{2} (T_\infty - T_1^i) + k \frac{l}{2} \frac{T_2^i - T_1^i}{l} + k \frac{l}{2} \frac{T_4^i - T_1^i}{l} + \dot{g}_0 \frac{l^2}{4} = \rho \frac{l^2}{4} C \frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$\text{Node 2: } hl(T_\infty - T_2^i) + k \frac{l}{2} \frac{T_1^i - T_2^i}{l} + k \frac{l}{2} \frac{T_3^i - T_2^i}{l} + kl \frac{T_5^i - T_2^i}{l} + \dot{g}_0 \frac{l^2}{2} = \rho \frac{l^2}{2} C \frac{T_2^{i+1} - T_2^i}{\Delta t}$$

$$\text{Node 3: } hl(T_\infty - T_3^i) + k \frac{l}{2} \frac{T_2^i - T_3^i}{l} + k \frac{l}{2} \frac{T_6^i - T_3^i}{l} + \dot{g}_0 \frac{l^2}{4} = \rho \frac{l^2}{4} C \frac{T_3^{i+1} - T_3^i}{\Delta t}$$

$$\left(\text{It can be rearranged as } T_3^{i+1} = \left(1 - 4\tau - 4\tau \frac{hl}{k} \right) T_3^i + 2\tau \left(T_4^i + T_6^i + 2 \frac{hl}{k} T_\infty + \frac{\dot{g}_0 l^2}{2k} \right) \right)$$

$$\text{Node 4: } \dot{q}_L l + k \frac{l}{2} \frac{T_1^i - T_4^i}{l} + k \frac{l}{2} \frac{140 - T_4^i}{l} + kl \frac{T_5^i - T_4^i}{l} + \dot{g}_0 \frac{l^2}{2} = \rho \frac{l^2}{2} C \frac{T_4^{i+1} - T_4^i}{\Delta t}$$

$$\text{Node 5 (interior): } T_5^{i+1} = (1 - 4\tau) T_5^i + \tau \left(T_2^i + T_4^i + T_6^i + 140 + \frac{\dot{g}_0 l^2}{k} \right)$$

Node

6:

$$hl(T_\infty - T_6^i) + k \frac{l}{2} \frac{T_3^i - T_6^i}{l} + kl \frac{T_5^i - T_6^i}{l} + kl \frac{140 - T_6^i}{l} + k \frac{l}{2} \frac{T_7^i - T_6^i}{l} + \dot{g}_0 \frac{3l^2}{4} = \rho \frac{3l^2}{4} C \frac{T_6^{i+1} - T_6^i}{\Delta t}$$

$$\text{Node 7: } hl(T_\infty - T_7^i) + k \frac{l}{2} \frac{T_6^i - T_7^i}{l} + k \frac{l}{2} \frac{T_8^i - T_7^i}{l} + kl \frac{140 - T_7^i}{l} + \dot{g}_0 \frac{l^2}{2} = \rho \frac{l^2}{2} C \frac{T_7^{i+1} - T_7^i}{\Delta t}$$

$$\text{Node 8: } h \frac{l}{2} (T_\infty - T_8^i) + k \frac{l}{2} \frac{T_7^i - T_8^i}{l} + k \frac{l}{2} \frac{140 - T_8^i}{l} + \dot{g}_0 \frac{l^2}{4} = \rho \frac{l^2}{4} C \frac{T_8^{i+1} - T_8^i}{\Delta t}$$

where $\dot{g}_0 = 2 \times 10^7 \text{ W/m}^3$, $\dot{q}_L = 8000 \text{ W/m}^2$, $l = 0.015 \text{ m}$, $k = 15 \text{ W/m} \cdot ^\circ\text{C}$, $h = 80 \text{ W/m}^2 \cdot ^\circ\text{C}$, and $T_\infty = 25^\circ\text{C}$.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 8 equations above is the coefficient of T_3^i in the T_3^{i+1} expression since it is exposed to most convection per unit volume (this can be verified), and thus the stability criteria for this problem can be expressed as

$$1 - 4\tau - 4\tau \frac{hl}{k} \geq 0 \rightarrow \tau \leq \frac{1}{4(1 + hl/k)} \rightarrow \Delta t \leq \frac{l^2}{4\alpha(1 + hl/k)}$$

since $\tau = \alpha \Delta t / l^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \leq \frac{(0.015 \text{ m})^2}{4(3.2 \times 10^{-6} \text{ m}^2/\text{s})[1 + (80 \text{ W/m}^2 \cdot ^\circ\text{C})(0.015 \text{ m})/(15 \text{ W/m} \cdot ^\circ\text{C})]} = 16.3 \text{ s}$$

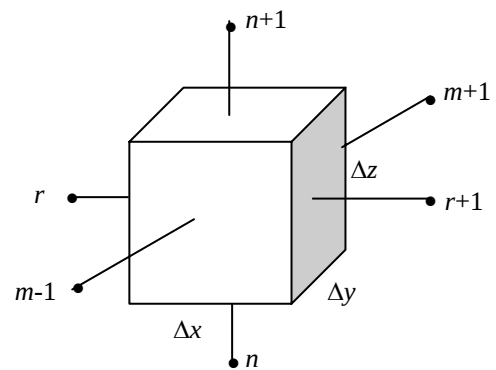
Therefore, any time step less than 16.3 s can be used to solve this problem. For convenience, we choose the time step to be $\Delta t = 15 \text{ s}$. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha \Delta t}{l^2} = \frac{(3.2 \times 10^{-6} \text{ m}^2/\text{s})(15 \text{ s})}{(0.015 \text{ m})^2} = 0.2133 \quad (\text{for } \Delta t = 15 \text{ s})$$

Using the specified initial condition as the solution at time $t = 0$ (for $i = 0$), sweeping through the 9 equations above will give the solution at intervals of 15 s. Using a computer, the solution at the upper corner node (node 3) is determined to be **441, 520, and 529°C** at 2, 5, and 30 min, respectively. It can be shown that the steady state solution at node 3 is 531°C.

5-107 Starting with an energy balance on a volume element, the three-dimensional transient *explicit* finite difference equation for a general interior node in rectangular coordinates for $T(x, y, z, t)$ for the case of constant thermal conductivity k and no heat generation is to be obtained.

Analysis We consider a rectangular region in which heat conduction is significant in the x and y directions. There is no heat generation in the medium, and the thermal conductivity k of the medium is constant. Now we divide the x - y - z region into a *mesh* of nodal points which are spaced Δx , Δy , and Δz apart in the x , y , and z directions, respectively, and consider a general interior node (m, n, r) whose coordinates are $x = m\Delta x$, $y = n\Delta y$, and $z = r\Delta z$. Noting that the volume element centered about the general interior node (m, n, r) involves heat conduction from six sides (right, left, front, rear, top, and bottom) and expressing them at previous time step i , the transient explicit finite difference formulation for a general interior node can be expressed as



$$\begin{aligned}
& k(\Delta y \times \Delta z) \frac{T_{m-1,n,r}^i - T_{m,n,r}^i}{\Delta x} + k(\Delta x \times \Delta z) \frac{T_{m,n+1,r}^i - T_{m,n,r}^i}{\Delta y} + k(\Delta y \times \Delta x) \frac{T_{m+1,n,r}^i - T_{m,n,r}^i}{\Delta z} \\
& + k(\Delta x \times \Delta y) \frac{T_{m,n-1,r}^i - T_{m,n,r}^i}{\Delta x} + k(\Delta x \times \Delta y) \frac{T_{m,n,r-1}^i - T_{m,n,r}^i}{\Delta y} + k(\Delta x \times \Delta y) \frac{T_{m,n,r+1}^i - T_{m,n,r}^i}{\Delta z} \\
& = \rho(\Delta x \times \Delta y \times 1)C \frac{T_{m,n}^{i+1} - T_{m,n}^i}{\Delta t}
\end{aligned}$$

Taking a cubic mesh ($\Delta x = \Delta y = \Delta z = l$) and dividing each term by k gives, after simplifying,

$$T_{m-1,n,r}^i + T_{m+1,n,r}^i + T_{m,n+1,r}^i + T_{m,n-1,r}^i + T_{m,n,r-1}^i + T_{m,n,r+1}^i - 6T_{m,n,r}^i = \frac{T_{m,n,r}^{i+1} - T_{m,n,r}^i}{\tau}$$

where $\alpha = k / (\rho C)$ is the thermal diffusivity of the material and $\tau = \alpha \Delta t / l^2$ is the dimensionless mesh Fourier number. It can also be expressed in terms of the temperatures at the neighboring nodes in the following easy-to-remember form:

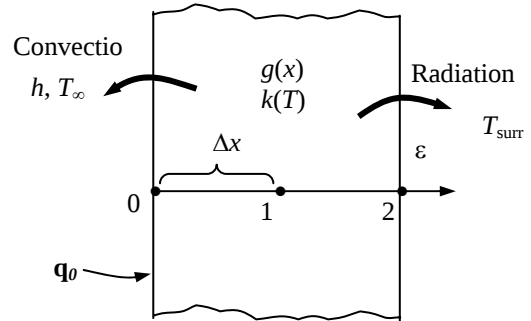
$$T_{\text{left}}^i + T_{\text{top}}^i + T_{\text{right}}^i + T_{\text{bottom}}^i + T_{\text{front}}^i + T_{\text{back}}^i - 6T_{\text{node}}^i = \frac{T_{\text{node}}^{i+1} - T_{\text{node}}^i}{\tau}$$

Discussion We note that setting $T_{\text{node}}^{i+1} = T_{\text{node}}^i$ gives the steady finite difference formulation.

5-109 A plane wall with variable heat generation and variable thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ and convection at the left (node 0) and radiation at the right boundary (node 2). The explicit transient finite difference formulation of the problem using the energy balance approach method is to be determined.

Assumptions 1 Heat transfer through the wall is given to be transient, and the thermal conductivity and heat generation to be variables. 2 Heat transfer is one-dimensional since the plate is large relative to its thickness. 3 Radiation from the left surface, and convection from the right surface are negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* finite difference formulations become



Left boundary node (node 0):
$$k_0^i A \frac{T_1^i - T_0^i}{\Delta x} + \dot{q}_0 A + hA(T_\infty - T_0^i) + \dot{g}_0^i (A\Delta x / 2) = \rho A \frac{\Delta x}{2} C \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Interior node (node 1):
$$k_1^i A \frac{T_0^i - T_1^i}{\Delta x} + k_1^i A \frac{T_2^i - T_1^i}{\Delta x} + \dot{g}_1^i (A\Delta x) = \rho A \Delta x C \frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Right boundary node (node 2):

$$k_2^i A \frac{T_1^i - T_2^i}{\Delta x} + \varepsilon \sigma A [(T_{surr}^i + 273)^4 - (T_2^i + 273)^4] + \dot{g}_2^i (A\Delta x / 2) = \rho A \frac{\Delta x}{2} C \frac{T_2^{i+1} - T_2^i}{\Delta t}$$